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business cycle: Extending the Smooth
Transition framework**

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Monetary policy shocks over the business cycle: Extending the Smooth Transition framework*

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Abstract

We extend the Smooth Transition Vector Autoregressive model to allow for identification via a combination of external instruments and sign restrictions, while estimating rather than calibrating the parameters ruling the nonlinearity of the model. We hence offer an alternative to using the recursive identification with selected calibrated parameters, which is the main approach currently available. We use the model to study how the effects of monetary policy shocks change over the business cycle. We show that financial variables, inflation and output respond to a monetary shock more in a recession than in an expansion, in line with the predictions from the financial accelerator literature.

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Keywords: Nonlinear models, proxy SVARs, monetary policy shocks, sign restrictions.

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1 Introduction

Does the effectiveness of economic policy depend on the economic conditions prevailing when the policy maker intervenes? The profession has reflected on this question since the early days of the business cycle literature ([Mitchell and Gay, 1927](#)). A large number of studies investigates if the effects of monetary or fiscal policy depend on the phase of the business cycle, on the prevailing credit conditions, on the level of uncertainty, or on others measures (see e.g. [Weise, 1999](#), [Auerbach and Gorodnichenko, 2013](#), [Balke, 2000](#) and [Pellegrino, forthcoming](#)). The common feature of these and other contributions is the use of empirical models that explicitly model nonlinearities in the data. These models raise technical challenges, requiring the researcher to make delicate decisions about the model specification and structural identification.

The first contribution of the paper is to extend the scope of a popular nonlinear time series model, the Smooth Transition Vector Autoregressive (STVAR).¹ The literature on STVAR models typically identifies the structural shocks of interest using recursive exclusion restrictions, and often calibrates rather than estimates the parameters pinning down the nonlinearity of the model (for instance [Auerbach and Gorodnichenko, 2012](#)). We address both limitations and propose a tractable Bayesian STVAR model with two features. First, it allows for identification of the shocks of interest via external instruments, potentially complementing their identification with sign restrictions to sharpen inference. Second, it estimates the key parameters specifying the model's nonlinearity, requiring no more than a standard, low-dimensional Metropolis-Hastings algorithm. The model hence offers a rich and tractable framework to address a variety of research questions, and lends itself to the computation of nonlinear impulse responses in the spirit of [Koop et al. \(1996\)](#).

¹Smooth transition models were independently introduced by [Bacon and Watts \(1971\)](#), [Goldfeld and Quandt \(1972\)](#) and [Maddala \(1977\)](#) in univariate regression models. They were then extended to univariate autoregressive models by [Chan and Tong \(1986\)](#) and [Teräsvirta \(1994\)](#) and made popular in a multivariate framework by [Auerbach and Gorodnichenko \(2012\)](#). For a discussion of the smooth transition model, see [Granger and Teräsvirta \(1993\)](#), [Teräsvirta et al. \(2010\)](#) and [Dijk et al. \(2002\)](#).

The second contribution of the paper is to document that monetary policy shocks have a stronger effect on the economy in a recession than in an expansion. We apply our methodology to extend the linear model by [Gertler and Karadi \(2015\)](#) and [Jarociński and Karadi \(2020\)](#) to a nonlinear setting, and identify a monetary shock using a combination of a high-frequency external instrument and a set of mild sign restrictions. We then simulate a monetary shock that either decreases the interest rate by 25 basis points in a recession, or increases it by the same amount in an expansion. We show that financial variables are considerably more responsive to the shock in a recession than in an expansion. The spread by [Gilchrist and Zakrajšek \(2012\)](#) falls by 40 basis points with a monetary expansion in recession, and increases by 8 basis points with a monetary contraction in an expansion. Similarly, the S&P500 increases by 7.7% in the first scenario and decreases by 2.3% in the second scenario. We then document that both inflation and output respond more in a recession. The difference in the response of real GDP equals 0.2 percentage points, comparable in size to the effect on real GDP usually found in linear models. We show that this nonlinearity is driven by whether the shock is given in a recession or in an expansion, and not by whether the shock is expansionary or contractionary.

Our paper is part of the effort to extend the identification schemes available within STVAR models beyond the recursive identification. To the best of our knowledge, the first attempt in this direction is due to earlier versions of [Bolboaca and Fischer \(2019\)](#), who extend the STVAR framework to identification with restrictions on the forecast error decomposition. Contrary to their work, we aim for identification via external instruments (with or without additional sign restrictions) in a Bayesian framework. The tractability of our framework builds on the fact that we use external instruments as [Paul \(2020\)](#) does in a time-varying coefficient VAR model à la [Primiceri \(2005\)](#), namely as exogenous regressors. We extend the approach by [Paul \(2020\)](#) to smooth transition models and complement it to allow for sign restrictions on the same shocks. Recently, [Carriero et al. \(2018\)](#) proposed using external instruments in a Multivariate

Autoregressive Index VAR model with a smooth transition. We view our approach as complementary but less restrictive, because we do not restrict contemporaneous impulse responses to change along regimes only by a scalar.

What makes our approach computationally efficient and easy to implement is the fact that we derive the marginal (rather than the conditional) posterior distribution of the key parameters ruling the nonlinearity of the model. This procedure allows us to isolate the computationally challenging part of the problem and to employ a standard Metropolis-Hastings algorithm, combined with direct sampling procedures. Visual inspection of the marginal posterior can then aid the researcher by uncovering potential posterior sampling challenges even before running the algorithm. The computational advantage of working with the marginal distribution has been exploited in univariate applications of smooth transition models, see [Bacon and Watts \(1971\)](#) and [Lubrano \(2000\)](#). However, multivariate extensions have built posterior sampling on the conditional posterior distribution, hence requiring a Metropolis-Hastings-within-Gibbs sampler (as in [Gefang and Strachan, 2010](#), [Gefang, 2012](#), [Carriero et al., 2018](#)). Monte Carlo simulations document that our approach works successfully in recovering the true impulse responses.

Smooth transition nonlinearities have also been used in Local Projection models, as for instance in [Auerbach and Gorodnichenko \(2013\)](#) and [Tenreyro and Thwaites \(2016\)](#). While it has been shown that VAR and LP models can detect the same estimands asymptotically in a linear setting ([Dufour and Renault, 1998](#), [Plagborg-Møller and Wolf, 2021](#)), this equivalence does not necessarily hold in nonlinear models (see, for instance, [Gonçalves et al., 2021](#)). We focus our analysis on the VAR framework, building on the insight by [Lubrano \(2000\)](#). We are not aware of a literature that estimates (rather than calibrates) the parameters specifying the nonlinearity in LP smooth transition models.

The results of our application are in line with [Kakes \(1998\)](#), [Peersman and Smets \(2002\)](#), [Dolado and Dolores \(2001\)](#), [Garcia and Schaller \(2002\)](#), and [Lo and Piger](#)

(2005), who study the nonlinear effects of monetary policy during recessions and expansion. They use univariate or multivariate Markov Switching regressions, either under the restrictive assumption that the interest rate is exogenous to the economy, or employing the timing restrictions from the recursive identification. Our results are consistent with the Threshold VAR analysis by [Atanasova \(2003\)](#), who uses a recursive assumption. Within the literature that employs a smooth transition type of nonlinearity, our results are in line with [Weise \(1999\)](#). Yet, his result is hard to interpret because it predicts that an exogenous increase in money supply has contractionary effects on output. Our results partly call into question the results by [Tenreyro and Thwaites \(2016\)](#), who find that monetary shocks have a stronger effect in expansions. Part of their nonlinear effect on output is associated with an “output puzzle”, namely a short term increase in output in response to a monetary contraction during recessions (see also [Miranda-Agrippino and Ricco, 2021](#), for a discussion). [Tenreyro and Thwaites \(2016\)](#) document that in a recession, the Excess Bond Premium by [Gilchrist and Zakrajšek \(2012\)](#) increases more than in an expansion, a result consistent with our finding that monetary shocks have a stronger effect during recessions.²

Our results support the prediction of the literature on the financial accelerator ([Blinder, 1987](#), [Bernanke and Gertler, 1989](#), [Kiyotaki and Moore, 1997](#), [Gertler and Hubbard, 1988](#)). As highlighted in this literature, in a recession the quality of balance sheets is poor, borrowers are more dependent on external finance, and borrowing constraints are more likely to bind. Accordingly, the general equilibrium effects on income and profits associated with a tightening or loosening of monetary policy have a stronger effect on borrowing and economic activity. The results suggest that monetary policy is effective at the times when it is needed the most to sustain output. Complementary interpretations of the possible nonlinearity in the effects of monetary

²Our paper does not directly address two other type of nonlinearities, namely if small or large shocks have proportional effects, and if positive and negative shocks have the same effects, up to the sign of the response. See, for instance, [Cover \(1992\)](#), [Morgan \(1993\)](#), [Ravn and Sola \(2004\)](#), [Karras \(2013\)](#), [Debortoli et al. \(2020\)](#).

policy shocks build on time varying price flexibility (Vavra, 2014, Berger and Vavra, 2014) or on agents' loss aversion (Santoro et al., 2014).

The paper is organized as follows. Section 2 outlines the model and discusses inference in a general framework. Section 3 shows the application to monetary policy shocks. Section 4 concludes.

2 The model

The tractability of our multivariate smooth transition framework owes to the particular combination of the specification of the model and the selection of prior beliefs. We discuss these key points in this section, and briefly illustrate the posterior sampler. Further details are provided in the Online Appendix.

2.1 The smooth transition structural VAR model

The model is given by

$$\mathbf{y}_t = g_{t-1} \cdot (\Pi_a \mathbf{x}_{t-1} + B_a \mathbf{m}_t + D_a \mathbf{q}_t) + \quad (1a)$$

$$+ (1 - g_{t-1}) \cdot (\Pi_b \mathbf{x}_{t-1} + B_b \mathbf{m}_t + D_b \mathbf{q}_t) + \mathbf{u}_t,$$

$$\mathbf{u}_t \sim N(\mathbf{0}, \Sigma_t), \quad (1b)$$

$$\Sigma_t = h_{t-1} \cdot \Sigma, \quad (1c)$$

$$g_{t-1} = \frac{1}{1 + e^{-\gamma(z_{t-1} - c)}}, \quad \gamma > 0, \quad (1d)$$

$$h_{t-1} = g_{t-1} + e^\psi \cdot (1 - g_{t-1}), \quad (1e)$$

$$\begin{cases} \psi = 0 & \text{if homoskedastic,} \\ \psi \neq 0 & \text{if heteroskedastic.} \end{cases}$$

The $k \times 1$ vector \mathbf{y}_t contains the endogenous variables of the model. The $kp \times 1$ vector $\mathbf{x}_{t-1} = (\mathbf{y}_{t-1}, \dots, \mathbf{y}_{t-p})'$ contains the p lags of the variables of the model. The

$k_m \times 1$ vector \mathbf{m}_t includes observable external instruments for the structural shocks of interest. The $k_q \times 1$ vector \mathbf{q}_t potentially contains control variables, a constant, and external measures of other structural shocks. \mathbf{m}_t and \mathbf{q}_t are exogenous in the model. $g(z_{t-1}, \gamma, c)$ indicates a scalar transition logistic function, which can evolve endogenously in the model depending on the specification of the transition variable z_{t-1} . $h(z_{t-1}, \gamma, c, \psi)$ indicates a scalar function modelling heteroskedasticity, where $\psi = 0$ implies homoskedasticity. Model (1) is effectively a time-varying linear combination of two latent processes,

$$\mathbf{y}_t = \Pi_a \mathbf{x}_{t-1} + B_a \mathbf{m}_t + D_a \mathbf{q}_t + \mathbf{u}_t, \quad (2a)$$

$$\mathbf{y}_t = \Pi_b \mathbf{x}_{t-1} + B_b \mathbf{m}_t + D_b \mathbf{q}_t + \mathbf{u}_t, \quad (2b)$$

which are associated with the extreme values $g(z_{t-1}, \gamma, c) = 1$ and $g(z_{t-1}, \gamma, c) = 0$. As the transition variable z_{t-1} evolves over time, the logistic transition function $g(z_{t-1}, \gamma, c)$ varies and implies a different point within the continuum of linear combinations of models (2a) and (2b) (Teräsvirta et al., 2014, Dijk et al., 2002).

Table 1: Trade-offs in modelling smooth transitions in a VAR model

| | If structural shocks of interest enter via | |
|------------------------------------------------------|--------------------------------------------------------------|-----------------------------------------------------------|
| | \mathbf{u}_t (innovations) | \mathbf{m}_t (regressors) |
| $\Sigma_t = \Sigma$ | nonlin-IRFs for only $h \geq 1$ computationally tractable | nonlin-IRFs for $h \geq 0$ computationally tractable |
| $\Sigma_t = g_{t-1}\Sigma_a + (1 - g_{t-1})\Sigma_b$ | nonlin-IRFs for $h \geq 0$ computationally challenging | nonlin-IRFs for $h \geq 0$ computationally challenging |
| $\Sigma_t = h_{t-1}\Sigma$ | | nonlin-IRFs for $h \geq 0$ computationally tractable |

Notes: ‘nonlin-IRFs’ stands for nonlinear impulse responses, while h represents the horizon of the impulse response.

Model (1) includes instruments for structural shocks as regressors $(\mathbf{m}_t, \mathbf{q}_t)$, and

features reduced form innovations that are either homoskedastic or heteroskedastic via the time-varying scalar h_{t-1} . It is these two features that keep the model tractable despite allowing for identification via external instruments and sign restrictions, as well as for the estimation rather than calibration of (γ, c) . To appreciate why, note that extending the classical linear VAR of the type $\mathbf{y}_t = \Pi \mathbf{x}_{t-1} + \mathbf{u}_t$ to a smooth transition framework poses a trade-off on how to model the innovations of the model. Assume, for the moment, that the structural shocks of interest are still a function of the innovations of the model, as from the middle column of [Table 1](#). A homoskedastic model ($\Sigma_t = \Sigma$) simplifies inference considerably, but assumes that impulse responses are the same on impact irrespectively of when the shock is generated. Several contributions have hence introduced heteroskedasticity of the type $\Sigma_t = g_{t-1}\Sigma_a + (1 - g_{t-1})\Sigma_b$ to generate nonlinear impulse responses already on impact. Yet, this modelling choice makes inference challenging, leading many to simplify the analysis by either calibrating rather than estimating (γ, c) or limiting the scope of the identification strategy (for instance using the recursive identification).³ However, as shown by [Paul \(2020\)](#) in the context of time-varying coefficient VAR models à la [Primiceri \(2005\)](#), the identification approach using external instruments ([Mertens and Ravn, 2013](#) and [Stock and Watson, 2012](#)) simplifies considerably when one introduces external instruments as regressors to the model, last column of [Table 1](#). In fact, impulse responses generated via \mathbf{m}_t are nonlinear already on impact without relying on the heteroskedasticity of the model.

We extend the approach by [Paul \(2020\)](#) to smooth transition models. We proceed

³Within the literature on smooth transition multivariate models, [Gefang and Strachan \(2010\)](#) assume $\Sigma_t = \Sigma$ and impose that impulse responses differ across regimes only after at least one period from when the shock is generated. [Auerbach and Gorodnichenko \(2012\)](#) and [Caggiano et al. \(forthcoming\)](#) allow for time varying impulse responses already on impact, but calibrate (γ, c) and use a recursive identification scheme. [Carriero et al. \(2018\)](#) jointly estimate $(\Sigma_a, \Sigma_b, \gamma, c)$ and use identification via external instruments, but at the cost of assuming that impulse responses differ on impact across time only up to a scalar implied by the time-varying matrix Σ_t . The impact effect of structural shocks is frequently assumed constant across regimes on impact also in related literatures, see for instance [de Dios Tena and Tremayne \(2009\)](#) in the Threshold VAR model, or [Aastveit et al. \(2017\)](#) and [Pellegrino \(forthcoming\)](#) in Interacted VAR models. See also [Balke \(2000\)](#) and [Galvao and Marcellino \(2014\)](#) for Threshold VAR models that allow for a nonlinear impact effects of the structural shocks but within the recursive identification.

in two complementary ways to avoid assuming that only the structural shock(s) of interest in \mathbf{m}_t potentially generate nonlinear responses already on impact. First, we control for instruments of other shocks in \mathbf{q}_t . Second, if needed, we also allow for heteroskedasticity in the innovations, limiting the form of heteroskedasticity to a case that facilitates posterior sampling ($\Sigma_t = h_{t-1} \cdot \Sigma$). Monte Carlo simulations suggest that a model with homoskedastic errors on data simulated under heteroskedasticity can still recover the true impulse responses of interest even when additional controls are not included (see [Appendix D](#) of the Online Appendix). For this reason, our preferred specification features homoskedastic shocks, which makes inference particularly tractable.

2.2 Inference

As widely discussed, for instance, in [Lubrano \(2001\)](#), [Dijk et al. \(2002\)](#), [Gerlach and Chen \(2008\)](#), and [Livingston and Nur \(2017\)](#), inference in smooth transition models is made challenging by the way in which (γ, c) affect the transition variable and the likelihood function of the model. For $\gamma = 0$, $g(z_{t-1}, \gamma, c) = 0.5$ for any value of z_{t-1} , making (Π_a, B_a, D_a) not separately identified from (Π_b, B_b, D_b) . Values of γ in the neighbourhood of zero make the parameters $(\Pi_a, B_a, D_a, \Pi_b, B_b, D_b)$ weakly identified, raising problems in the routines for numerical maximization. As γ increases, $g(z_{t-1}, \gamma, c)$ approaches the indicator function $I[z_{t-1} > c]$, and becomes very steep around the inflection point $z_{t-1} = c$ already for relatively low values of γ . This, in turn, makes any difference in high but different values of γ hard to detect. We provide a visual illustration of these problems in the Online Appendix.

We follow [Lubrano \(2001\)](#), [Lopes and Salazar \(2006\)](#) and [Livingston and Nur \(2017\)](#) and address the above challenge using a Bayesian approach. The parameters of the model are $(\boldsymbol{\theta}, \Sigma, \gamma, c, \psi)$, with $\boldsymbol{\theta} = \text{vec}([\Pi_a, B_a, D_a, \Pi_b, B_b, D_b])$ and $\text{vec}(\cdot)$ the operator that stacks the columns of a matrix vertically. Conditioning on (γ, c, ψ) , the model is linear. This makes it natural to restrict the prior on $(\boldsymbol{\theta}, \Sigma)$ to the conjugate Normal-

inverse-Wishart prior, conditioning on (γ, c, ψ) . By contrast, the prior on (γ, c, ψ) can be freely selected by the researcher. In short, we use prior beliefs

$$p(\boldsymbol{\theta}, \Sigma, \gamma, c, \psi) \propto \mathbf{I}\{\boldsymbol{\theta}\} \cdot \tilde{p}(\boldsymbol{\theta}|\Sigma, \gamma, c, \psi) \cdot \tilde{p}(\Sigma|\gamma, c, \psi) \cdot \tilde{p}(\gamma, c, \psi), \quad (3a)$$

$$\tilde{p}(\boldsymbol{\theta}|\Sigma, \gamma, c, \psi) = N(\boldsymbol{\mu}, V \otimes \Sigma), \quad (3b)$$

$$\tilde{p}(\Sigma|\gamma, c, \psi) = iW(S, d), \quad (3c)$$

$$\tilde{p}(\gamma, c, \psi) = \text{free}. \quad (3d)$$

The indicator function $\mathbf{I}\{\boldsymbol{\theta}\}$ equals one when $\boldsymbol{\theta}$ satisfies the restrictions of interest (if any), for instance restrictions on the stationarity of the model, or sign restrictions on the response to structural shocks.

The combination of the model specification from [Section 2.1](#) and prior beliefs from equation (3) makes inference particularly tractable, because the joint posterior distribution satisfies

$$p(\boldsymbol{\theta}, \Sigma, \gamma, c, \psi|Y) \propto \mathbf{I}\{\boldsymbol{\theta}\} \cdot \tilde{p}(\boldsymbol{\theta}|Y, \Sigma, \gamma, c, \psi) \cdot \tilde{p}(\Sigma|Y, \gamma, c, \psi) \cdot \tilde{p}(\gamma, c, \psi|Y), \quad (4a)$$

$$\tilde{p}(\boldsymbol{\theta}|Y, \Sigma, \gamma, c, \psi) = N(\boldsymbol{\mu}^*, \bar{V}^*(\gamma, c, \psi) \otimes \Sigma), \quad (4b)$$

$$\tilde{p}(\Sigma|Y, \gamma, c, \psi) = iW(S^*(\gamma, c, \psi), d^*), \quad (4c)$$

$$\tilde{p}(\gamma, c, \psi|Y) \propto \tilde{p}(\gamma, c, \psi) \cdot |\det(S^*(\gamma, c, \psi))|^{-\frac{d+T}{2}} \cdot |\det(\bar{V}^{-1} + W(\gamma, c)W(\gamma, c)')|^{-\frac{k}{2}}, \quad (4d)$$

with $(\boldsymbol{\mu}^*, \bar{V}^*, S^*, d^*)$ the standard results from linear VAR models, once we condition on (γ, c, ψ) . The crucial feature of the joint posterior is the analytical derivation of the marginal posterior $\tilde{p}(\gamma, c, \psi|Y)$ rather than the conditional posterior $\tilde{p}(\gamma, c, \psi|Y, \boldsymbol{\theta}, \Sigma)$. The joint posterior distribution $p(\boldsymbol{\theta}, \Sigma, \gamma, c, \psi|Y)$ can then be sampled in three main steps:

- 1) use a Metropolis-Hastings algorithm to sample from $\tilde{p}(\gamma, c, \psi|Y)$ (or from $\tilde{p}(\gamma, c|Y)$ under homoskedasticity);

- 2) conditioning on (γ, c, ψ) , draw $(\boldsymbol{\theta}, \Sigma)$ from $\tilde{p}(\Sigma|Y, \gamma, c, \psi)$ and $\tilde{p}(\boldsymbol{\theta}|Y, \Sigma, \gamma, c, \psi)$ using direct sampling;
- 3) store $(\boldsymbol{\theta}, \Sigma, \gamma, c, \psi)$ if $\boldsymbol{\theta}$ satisfies the restrictions modelled via the indicator function $I\{\boldsymbol{\theta}\}$.

We refer to the Online Appendix for a more detailed discussion of the algorithm.

The key feature that allows for both the tractable estimation of (γ, c) and the identification via external instruments and sign restrictions is the use of a Normal prior for $\boldsymbol{\theta}$ with a Kronecker structure on the variance (i.e. $\boldsymbol{\theta} \sim N(\boldsymbol{\mu}, V \otimes \Sigma)$), rather than introducing prior independence of $\boldsymbol{\theta}$ from Σ (i.e. $\boldsymbol{\theta} \sim N(\boldsymbol{\mu}, \tilde{V})$). In a linear VAR, allowing for prior independence is often desirable, because it is less restrictive and it still allows for efficient sampling via a Gibbs sampler (see for instance [Koop and Korobilis, 2010](#)). However, prior independence of $\boldsymbol{\theta}$ on Σ becomes computationally costly in a smooth transition model, where posterior sampling also involves (γ, c) , as well as ψ if the model is heteroskedastic. The Bayesian Smooth Transition VAR literature that estimates (γ, c) tends to use an independent Normal prior on $\boldsymbol{\theta}$, which implies sampling the joint posterior distribution using a Metropolis-Hastings-within-Gibbs sampler ([Gefang and Strachan, 2010](#), [Gefang, 2012](#) and [Carriero et al., 2018](#)). The Metropolis-Hastings step is required for $\tilde{p}(\gamma, c|Y, \boldsymbol{\theta}, \Sigma)$, a distribution which can be ill-shaped in a way that constantly changes as the chain progresses on $(\boldsymbol{\theta}, \Sigma)$. By contrast, imposing a Kronecker structure in the prior for $\boldsymbol{\theta}$, while more restrictive, allows for the analytical derivation of the *marginal* distribution $\tilde{p}(\gamma, c, \psi|Y)$ (or $\tilde{p}(\gamma, c|Y)$ under homoskedasticity), which is constant in $(\boldsymbol{\theta}, \Sigma)$. Hence, no Gibbs sampler is required. In a homoskedastic setting one can use visual inspection of $\tilde{p}(\gamma, c|Y)$ prior to running the algorithm to better tackle the potentially challenging shape of $\tilde{p}(\gamma, c|Y)$.⁴

⁴The computational convenience of building the posterior sampler on the marginal rather than the conditional posterior distribution of selected parameters is already fully acknowledged in threshold models, both in univariate and in multivariate specifications (see [Geweke and Terui, 1993](#) and [Forbes et al., 1999](#), respectively). When [Bacon and Watts \(1971\)](#) extended the threshold model to a univariate smooth transition model, they used the same insight, employing a preliminary visual

In short, the estimation approach discussed in this section has a variety of convenient features. First, it estimates all parameters jointly rather than either calibrate (γ, c, ψ) or estimate them using a two-step procedure based on a grid search (Teräsvirta et al., 2014). Second, it involves no more than a low-dimensional Metropolis-Hastings procedure for the numerically challenging part of the problem, given that conditioning on (γ, c, ψ) , direct sampling can be used for (θ, Σ) .⁵ Third, if needed, the identifying restrictions implicitly introduced via the external instruments can be complemented with restrictions on the contemporaneous effects of the shocks, on lagged values of the impulse responses, on other structural parameter of interest, or on the persistence properties of the process. Lastly, it allows for additional restrictions to separately identify more than one shock even if the external instruments do not only capture one shock of interest, as in Braun and Brüggemann (2017), Piffer and Podstawski (2018), Giacomini et al. (2021) and Arias et al. (2021). Note that while sign restrictions are not strictly needed for the sampler to work, an external instrument for each shock of interest is needed. Having sampled from the joint posterior distribution of the parameter of the model, the analysis lends itself to the study of nonlinear impulse responses via Monte Carlo integration (Koop et al., 1996).⁶ We discuss how we compute nonlinear impulse responses in Appendix C of the Online Appendix.

We conclude this section with a final remark on the inference for γ . As mentioned above, the model is not identified for values of γ close to zero. For γ close to zero, $W(\gamma, c)$ approaches singularity, pushing $|\det(\bar{V}^{-1} + W(\gamma, c)W(\gamma, c)')|$ in equation (4d) towards infinity. Contrary to Gefang and Strachan (2010), we found that this in-

inspection of the marginal posterior distribution of (γ, c) to guide posterior sampling. To the best of our knowledge, the literature on smooth transition models has followed this approach only within univariate models (see also Lubrano, 2000 and Bauwens et al., 2000, Chapter 8). Our paper extends this insight to a multivariate framework.

⁵The sampling procedure takes approximately 3 hours for the baseline specification of our application. The computational time drops to approximately 3 minutes if no sign restrictions are imposed, suggesting that the sampler is very efficient, especially in circumstances where the proxy itself is sufficient to achieve precise identification.

⁶We follow the suggestion by Kilian and Lütkepohl (2017) and use the expression ‘nonlinear impulse responses’ rather than generalized impulse responses, to avoid confusion with the responses proposed by Pesaran and Shin (1998).

convenient feature of the model cannot always be offset via the curvature introduced via $\tilde{p}(\gamma)$. We hence follow what [Lubrano \(2000\)](#) proposes in the context of univariate smooth transition models. We introduce prior dependence in $(\boldsymbol{\theta}, \gamma)$ and calibrate $V(\gamma)$ such that $\tilde{p}(\boldsymbol{\theta}|\Sigma, \gamma)$ becomes more informative as γ approaches zero. We discuss the details of this prior in [Appendix E](#) of the Online Appendix.

3 Monetary policy shocks and the business cycle

The model developed in [Section 2](#) is particularly suitable to study monetary policy shocks. In fact, both sign restrictions and external instruments are extensively used in the literature to identify monetary policy shocks (see, for instance, [Uhlig, 2005](#) and [Gertler and Karadi, 2015](#)). We apply the model from [Section 2](#) to revisit the analysis by [Gertler and Karadi \(2015\)](#) and [Jarociński and Karadi \(2020\)](#) in a nonlinear setting. We study how the effects of a monetary policy shock change depending on the phase of the business cycle at the time of the shock.

3.1 Data, model specification, priors

We introduce five monthly variables in the model, all on the US economy: the one-year government interest rate, the measure of spread computed by [Gilchrist and Zakrajšek \(2012\)](#) (which we will refer to as the GZ Spread), the log of the S&P500 index, the log of interpolated real GDP, and the log of the interpolated GDP deflator. We refer to [Appendix E](#) of the Online Appendix for the detailed discussion of the dataset. The selection of the variables follows [Jarociński and Karadi \(2020\)](#), except that we replace the GZ Excess Bond Premium with the GZ Spread to avoid using a variable that is the outcome of a linear estimation. The sample period starts in 1979M7, as in [Gertler and Karadi \(2015\)](#). We extend it until 2020M3, the month when the World Health Organization declared the COVID-19 pandemic. We include 12 lags in the model, as in [Gertler and Karadi \(2015\)](#) and [Jarociński and Karadi \(2020\)](#). The baseline

specification assumes homoskedasticity by setting $\psi = 0$.

Our nonlinear application requires specifying the transition variable z_{t-1} . We use a two month backward-looking moving average of the quarter-to-quarter log difference of real GDP (see [Appendix B](#) in the Online Appendix for the details). We construct the transition variable on quarter-to-quarter variations in order to better reflect the NBER timing of the recessions. We then select two as the length of the moving average process using the Deviance Information Criterion (DIC) by [Spiegelhalter et al. \(2002\)](#). By construction, higher values of the transition variable are associated with higher growth of real GDP, hence with a more expansionary phase of the business cycle. Combined with the restriction that γ is positive, we can view the linear models from equations (2a) and (2b) as the extreme cases for expansion and recession, respectively. The economy endogenously evolves within the continuum of models pinned down by all possible linear combinations of equations (2a) and (2b), shifting more towards (2a) whenever the average growth of real GDP increases.

Identification of the monetary shock is achieved using a combination of external instruments and mild sign restrictions. We use the instrument for the monetary policy shock computed by [Gertler and Karadi \(2015\)](#), which starts in 1990M2 and was updated by [Jarociński and Karadi \(2020\)](#) to cover the period until 2016M11.⁷ In principle, the instrument suffices to identify the model. In practice, we document that it leads to results qualitatively in line with theoretical predictions only in a linear version of the model, but not always in its nonlinear extension. This is not surprising, given that the effects of interest are now nonlinear and time-varying, hence more information may be required to identify the shocks of interest. For this reason, we complement the information from the instrument with mild sign restrictions on the conditionally linear impulse responses, i.e. the responses obtained when plugging the estimated values of the parameters from our model into the extreme cases from models (2a) and (2b).

We introduce identical sign restrictions in recessions and in expansion, hence en-

⁷The instrument in m_t is set equal to zero for the periods in which the instrument is not available. We do the same for the additional instruments introduced as controlled variables in \mathbf{q}_t .

suring that prior beliefs favour impulse responses that are invariant to the phase of the business cycle. In response to an increase in the external instrument by [Gertler and Karadi \(2015\)](#), we introduce the restriction that the one-year interest rate and the GZ Spread increase contemporaneously, and that the S&P500 decreases contemporaneously. In addition, we introduce the restriction that real GDP and the GDP deflator decrease for at least 2 periods within the first year and a half from the shock, irrespective of which periods, and even if the periods differ across regimes. The timing of the restrictions on the interest rate, the GZ Spread and the S&P500 are in line with large parts of the empirical and the theoretical literature. The weaker restrictions on the timing of the response of real GDP and the GDP deflator ensure that we allow for the results to potentially detect a different timing and persistence in the response of a monetary shock to output and inflation. These restrictions are introduced on the conditionally linear responses, which are a function of the parameters (Π_a, B_a, Π_b, B_b) .

Compared to the specification by [Gertler and Karadi \(2015\)](#), we control for additional measures of structural shocks in \mathbf{q}_t , in order not to limit the monetary shock to be the only structural shock that can contemporaneously affect the variables of the model in a nonlinear way even in the baseline homoskedastic case. We introduce the estimates of the TED spread shock by [Stock and Watson \(2012\)](#), the oil demand and oil supply shocks estimated by [Baumeister and Hamilton \(2019\)](#), and the instrument for uncertainty shocks by [Piffer and Podstawski \(2018\)](#).

[Appendix E](#) of the Online Appendix provides a detailed illustration of the prior distributions used. We follow common practice and standardize z_{t-1} to make (γ, c) scale-free. We start from a gamma prior for $\tilde{p}(\gamma)$, calibrating its hyperparameters to ensure an expected value equal to 2 and standard deviation equal to 3. We then truncate $\tilde{p}(\gamma)$ to $\gamma \geq 0.01$ to ensure that we stay away from the non-identified region, as also suggested by [Gefang and Strachan \(2010\)](#) and [Livingston and Nur \(2017\)](#). As for $\tilde{p}(c)$, we start from a Normal distribution centred at the median value of z_{t-1} , truncate it to have positive mass only between the 5th and 95th percentiles of z_{t-1} , and calibrate

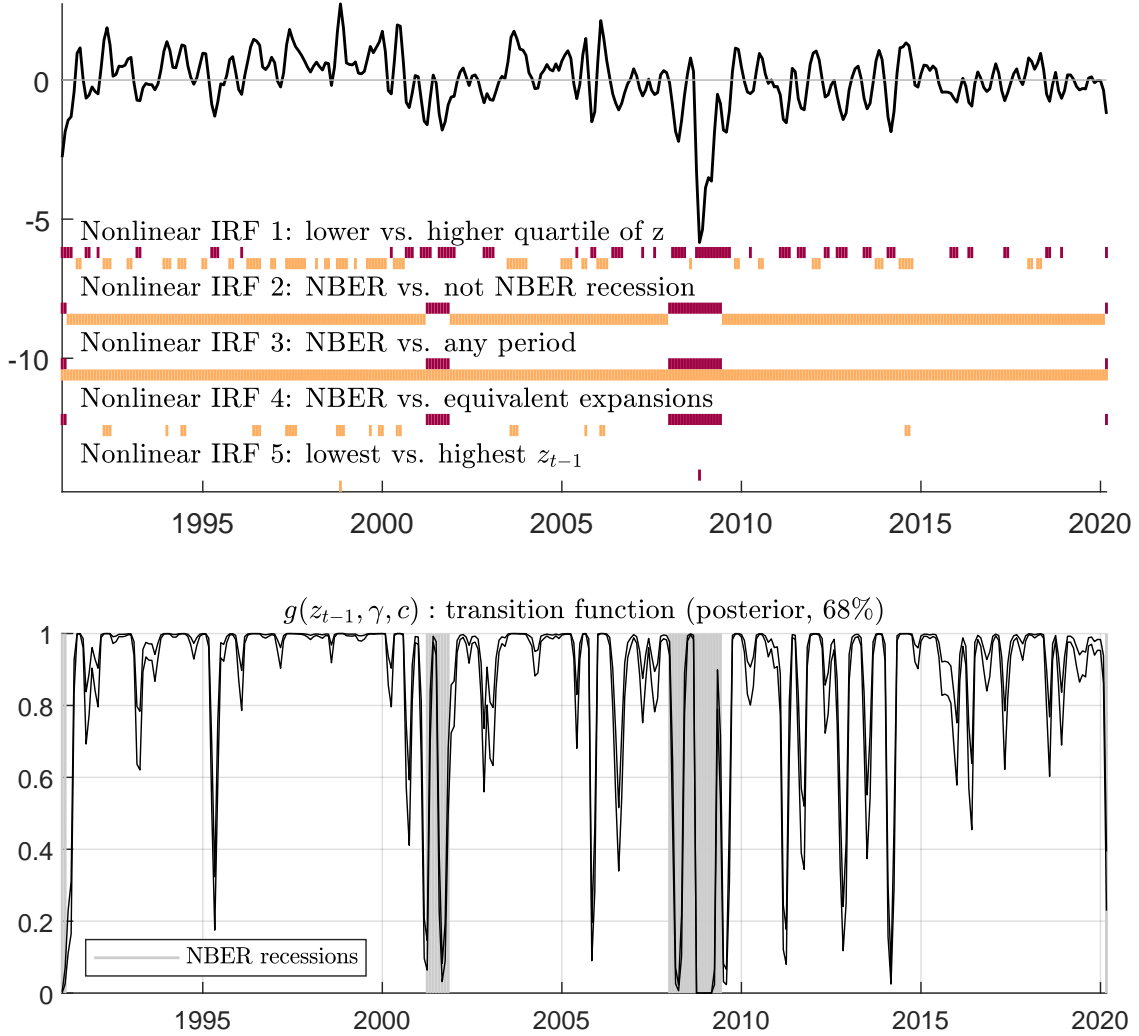
the variance to ensure that 95% probability mass of the truncated prior is between the 30th and 70th percentiles of z_{t-1} . We then set $\tilde{p}(\Sigma|\gamma, c) = \tilde{p}(\Sigma)$, calibrating the hyperparameters as in [Kadiyala and Karlsson \(1997\)](#). Lastly, we set $\tilde{p}(\boldsymbol{\theta}|\Sigma, \gamma, c) = \tilde{p}(\boldsymbol{\theta}|\Sigma, \gamma)$, calibrating the hyperparameters such that the prior is relatively uninformative conditioning on values of γ above 1 and progressively more informative as γ approaches zero. We use the indicator function $I\{\boldsymbol{\theta}\}$ from equation (3) to restrict the model to be stationary, and to introduce the sign restrictions for the monetary policy shock explained above in an accept-reject fashion. Since $\tilde{p}(\Sigma)$ requires a training sample, we employ the period 1979M7-1990M1 as a training sample and the remaining sample 1990M2-2020M3 as the estimation sample.

3.2 Results

We use the model to study if the impulse responses to a monetary policy shock differ depending on whether the central bank intervenes in a period closer to a recession or closer to an expansion. One way of addressing this question is to limit the analysis to conditionally linear impulse responses, which are the linear impulse responses obtained when plugging our estimates into models (2a) and (2b). We prefer not to build our analysis on conditionally linear impulse responses because the economy is hardly ever exactly in one of the two extreme regimes (2a) or (2b). In addition, even if it was, it could endogenously evolve away from it, a feature that must be taken into account. Instead, we build our analysis on nonlinear impulse responses in the spirit of [Koop et al. \(1996\)](#), and employ a battery of different definitions of recessions and expansions in order to study nonlinear effects across the business cycle more comprehensively.

We use the expressions “recession” and “expansion” to refer to different subperiods of the sample. Our first definition of the nonlinear impulse responses defines a recession as all periods in which the transition variable z_{t-1} is below the lowest quartile of all sample values of z_{t-1} , and defines an expansion using the highest quartile. Our second definition of nonlinear impulse responses groups periods according to the NBER

Figure 1: Smooth transition pattern
 z_{t-1} : transition variable



Note: See [Table F.2](#) and [Figure F.10](#) in the Online Appendix for the comparison to the prior distribution and for the number of months that belong to each definition of the nonlinear impulse responses.

business cycle dating. The third definition groups periods into NBER recession or any period of the sample, including the NBER recessions. The fourth definition compares the 29 periods of the sample classified as NBER recessions with the 29 periods in which z_{t-1} reached its highest 29 values. The last definition compares the two single periods associated with the highest and the lowest points of z_{t-1} . The top panel in

Figure 1 illustrates each classification of the periods, and plots the transition variable. The lower panel of Figure 1 shows the pointwise 68% band of the transition function. See also Table F.2 and Figure F.10 in the Online Appendix for a complementary illustration.

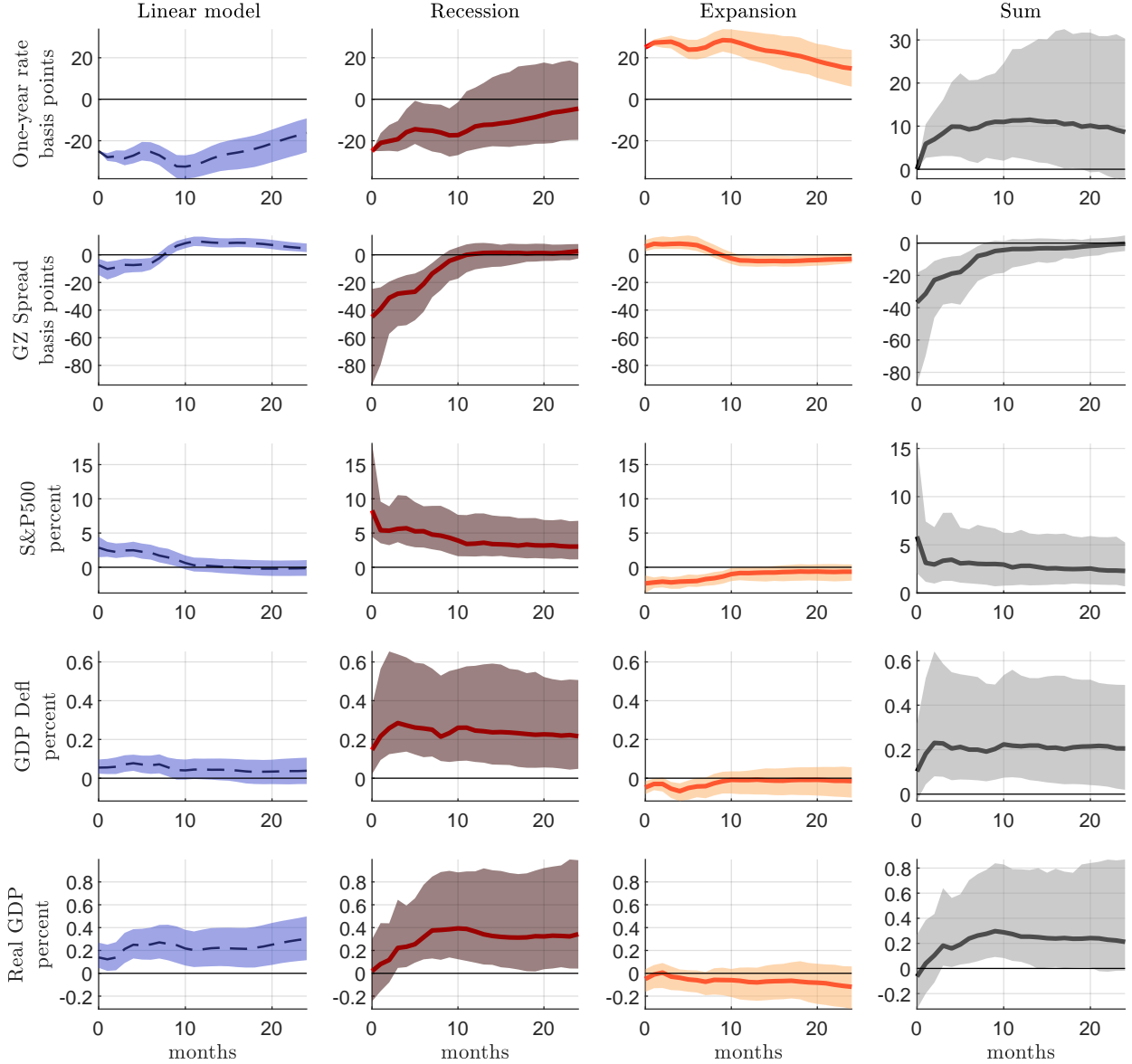
We explore the results of our analysis as follows. We first consider a linear setting, estimating model (2a) using the very same model and prior specification as from our nonlinear model.⁸ We then select a definition of recession and expansion and compute nonlinear impulse responses by simulating a monetary policy shock in either recession or in expansion, accounting for the fact that the model evolves endogenously after the shock. All monetary shocks are generated by scaling the appropriate entry of m_t in equation (1a) to decrease the interest rate on impact by 25 basis points in recession, or to increase it by the same amount in expansion. We do so to avoid discussing a less natural monetary expansion in an expansion, or a monetary contraction in a recession.⁹ The nonlinear impulse responses average out both the actual period at which the shock is simulated within the recessionary or expansionary subperiod, and the future shocks that hit the economy after the initial impact. Lastly, for each posterior draw, we compute the pointwise sum in the impulse responses between recessions and expansions in order to assess the statistical strength of possible nonlinearities. We refer the reader to Section A.3 of the Online Appendix for the specification and the diagnostics on the sampler, and to Appendix C of the Online Appendix for the detailed explanation of how we construct the nonlinear impulse responses.

The first column of Figure 2 shows the impulse responses associated with the linear model. It documents that an exogenous decrease in the interest rate by 25 basis points

⁸For the prior on θ , we use the same values of the hyperparameters that the nonlinear prior associates with values of γ above 1, see the discussion in Appendix E of the Online Appendix.

⁹As we document in Figure F.24-Figure F.25 of the Online Appendix, the nonlinearity documented in the analysis is driven by the different periods in which the shocks are generated, rather than by the fact that the generated shocks are of opposite sign. While the smooth transition model does not restrict positive and negative impulse responses to have identical effects up to the sign of the response, it also does not directly model this type of nonlinearity. See, instead, the models by Cover (1992) and Debortoli et al. (2020).

Figure 2: Posterior nonlinear impulse responses



Note: The figure shows pointwise median and 68% bands, using the first definition of the nonlinear impulse responses.

leads to an contemporaneous decrease of the GZ Spread by close to 8 basis points, and an increase in the S&500 index by around 3%. The shock generates inflation,

which materializes itself within the first six months from the shock. In addition, real GDP increases contemporaneously, but the strongest effects on output materialize around half a year from the shock, when output increases by around 0.25 percent. The results are qualitatively and quantitatively in line with [Gertler and Karadi \(2015\)](#) and [Jarociński and Karadi \(2020\)](#). In the linear model, the results are by construction the same (up to sign) following a monetary contraction rather than an expansion.

The remaining columns of [Figure 2](#) then show the results from the nonlinear model using the first definition of nonlinear impulse responses. An exogenous monetary expansion that decreases the interest rate by 25 basis points during a recession has a quantitatively strong impact effect on the financial variables, because the GZ Spread is found to fall by more than 40 basis points and the S&P500 increases by 7.7%. Inflationary pressures are strong relatively soon after the shock, but the effects on output do not materialize within the first few months. By contrast, a monetary contraction during an expansionary period generates impulse responses that are, in general, of the opposite sign as the ones in recession, but differ from them qualitatively and quantitatively. In an expansion, the monetary intervention generates effects on the GZ Spread that are more short lived and smaller in size, as the GZ Spread never increases by more than 8 basis points and the S&P500 never decreases by more than 2.3%. Output responds contemporaneously, but the effect on inflation is weaker.

The comparison of the pointwise marginal distributions from the middle columns in [Figure 2](#) suggests a sizeable nonlinearity in the response of the financial variables and the GDP deflator, but the results are less clear for the remaining variables. To further inspect the strength of the nonlinearities, the last column of [Figure 2](#) reports 68% bands on the marginal distributions of the pointwise sum in the impulse responses between recessions and expansions. The prior distributions imply a sum that is symmetric around zero, reflecting prior beliefs that there is no nonlinearity. The posterior distribution confirms the strength of the nonlinearity in the response of the GZ Spread and the S&P500, a nonlinearity that lasts for half a year. It also suggests a short lived

Table 2: Strength in the nonlinearities of the nonlinear impulse responses

| horizon → | 1 : 3 | 4 : 6 | 7 : 9 | 10 : 12 | 13 : 19 | 1 : 3 | 4 : 6 | 7 : 9 | 10 : 12 | 13 : 19 |
|-------------|------------------------|-------|-------|---------|---------|--------------|-------|-------|---------|---------|
| | One-year interest rate | | | | | GZ Spread | | | | |
| nonlinIRF 1 | .930 | .910 | .845 | .870 | .830 | .005 | .020 | .045 | .165 | .115 |
| nonlinIRF 2 | .845 | .815 | .735 | .730 | .735 | .005 | .030 | .100 | .260 | .180 |
| nonlinIRF 3 | .820 | .785 | .675 | .705 | .710 | .005 | .050 | .135 | .295 | .205 |
| nonlinIRF 4 | .870 | .810 | .740 | .760 | .730 | .005 | .035 | .130 | .280 | .225 |
| nonlinIRF 5 | .715 | .735 | .645 | .610 | .540 | .005 | .010 | .250 | .560 | .450 |
| | S&P500 | | | | | GDP deflator | | | | |
| nonlinIRF 1 | .935 | .935 | .930 | .915 | .895 | .760 | .940 | .940 | .945 | .915 |
| nonlinIRF 2 | .910 | .920 | .875 | .890 | .870 | .760 | .930 | .935 | .950 | .910 |
| nonlinIRF 3 | .885 | .890 | .875 | .845 | .845 | .760 | .935 | .930 | .925 | .895 |
| nonlinIRF 4 | .920 | .910 | .870 | .855 | .855 | .760 | .925 | .935 | .945 | .920 |
| nonlinIRF 5 | .890 | .920 | .850 | .780 | .800 | .760 | .945 | .955 | .960 | .950 |
| | Real GDP | | | | | | | | | |
| nonlinIRF 1 | .425 | .815 | .895 | .915 | .820 | | | | | |
| nonlinIRF 2 | .420 | .800 | .845 | .850 | .775 | | | | | |
| nonlinIRF 3 | .425 | .785 | .815 | .835 | .735 | | | | | |
| nonlinIRF 4 | .420 | .755 | .835 | .820 | .710 | | | | | |
| nonlinIRF 5 | .425 | .740 | .720 | .675 | .565 | | | | | |

Notes: For each variable of the model and for each definition of the nonlinear impulse responses, we select a set of the response horizons (from 1 to 3, from 4 to 6, and so on) and report the share of total posterior draws for which the sum of the impulse responses in recession and expansion is positive in each of the selected horizons. See [Figure F.14-Figure F.15](#) in the Online Appendix for the impulse responses of the definitions from 2 to 5 of the nonlinear impulse responses.

nonlinearity in the response of the one-year rate, illustrating that monetary policy remains expansionary during a recession for less than it remains contractionary during an expansion. This finding is consistent with the nonlinearity detected on the financial variables, which are arguably relevant for the response function of the central bank. The figure also suggests a strong nonlinearity in the response of the GDP deflator, which is found to respond more strongly in recession than in expansion. Finally, the figure shows that the monetary shock has stronger effects on output during the recessionary period, a nonlinearity that materializes three months after the shock and that disappears after a year from the shock. Quantitatively, the nonlinearity in the response of output of an exogenous 25 basis point variation in the interest rate amounts to around 0.20 percentage points, which is comparable to the estimate of the effect of

the shock in a linear model.

Table 2 complements the analysis by reporting the posterior probability that the sum of the impulse responses computed in the last column of Figure 2 is positive. A posterior probability strongly away from 0.50 suggests a nonlinearity. We first select a set of horizons of the impulse responses, for instance the first quarter from the shock, including the impact effect (this is the column titled 1:3). We then report the percentage of posterior draws for which the sum of the recessionary and expansionary impulse responses are above zero for all horizons within this set. We do the analysis for different periods of the impulse responses, and for all definitions of the nonlinear impulse responses used. Figure F.14-Figure F.15 in the Online Appendix report the associated impulse responses.

As documented in Table 2, we attach more than a 90% posterior probability that the GZ Spread is more responsive in a recession than in an expansion. This nonlinearity is particularly strong on impact. The nonlinearity on the S&P500 lasts longer, with a posterior probability that it responds more in recession than in an expansion of around 80%. While weaker, we find evidence that indeed the interest rate reverts back to zero faster in a recession. Lastly, there is approximately an 90% and 80% posterior probability that the GDP deflator and the real GDP respond more in a recession, respectively. This difference is mainly detected within 4 to 12 months from the shock, and lasts even longer for the GDP deflator.

In Appendix F of the Online Appendix we document that the results of the analysis are robust to a wide range of alternative model and prior specifications, including the use of a heteroskedastic model.

3.3 Comparing our results to the literature

Several contributions in the literature employ a STVAR model, but calibrate rather than estimate (γ, c) (for instance Auerbach and Gorodnichenko, 2012). To assess the possible implications of this approach, we compare our baseline results to the case in

which (γ, c) are calibrated rather than estimated. We set $c = 0$, as also in [Auerbach and Gorodnichenko \(2012\)](#). We then consider the arbitrary calibration of $\gamma = 4$ and $\gamma = 5$. The analysis, reported in [Appendix F](#) of the Online Appendix, shows that the results of a STVAR model can be quite sensitive to whether parameters are calibrated or estimated. When calibrating the parameters, we find that our result on the effect of output changes its sign, suggesting a stronger effect of monetary policy shock in expansion rather than recession. This suggests that it is indeed important to estimate rather than calibrate such parameters.

Other existing contributions have investigated whether monetary policy shocks have different effects on the economy depending on whether they occur at different stages of the business cycle. While the vast majority reports our same result, namely that monetary policy is more effective during periods of recession than expansion, we stress that several existing contributions are sometimes hard to reconcile with economic theory or rely on strong modelling assumptions. For instance, the results by [Weise \(1999\)](#) document that an exogenous increase in money supply leads to a contraction in output. The contributions by [Kakes \(1998\)](#), [Peersman and Smets \(2002\)](#), [Dolado and Dolores \(2001\)](#), [Garcia and Schaller \(2002\)](#), and [Lo and Piger \(2005\)](#) either introduce the assumption that the interest rate is exogenous to the economy, or identify the shocks of interest using timing restrictions.

To the best of our knowledge, the only contribution that documents an opposite result compared to us within a smooth transition framework is [Tenreyro and Thwaites \(2016\)](#).¹⁰ They too study if monetary policy becomes more or less effective in times of recession, and argue that monetary policy is less effective in a recession. The differences in the modelling strategy between our paper and theirs do not make it straightforward to compare the two findings. [Tenreyro and Thwaites \(2016\)](#) use a smooth transition

¹⁰[Alpanda et al. \(2019\)](#) find results that are similar to [Tenreyro and Thwaites \(2016\)](#) but in a threshold (rather than smooth transition) local projection models. They calibrate rather than estimate the key parameters ruling the nonlinearity in the model, and extend the analysis to a panel dimension.

local projection approach on quarterly data from 1967Q1 to 2002Q4 and calibrate (γ, c) . We use a smooth transition VAR model on monthly data from 1979M7 to 2020M3 and estimate (γ, c) . They identify the monetary shocks using the shocks by [Romer and Romer \(2004\)](#), while we use the shocks by [Gertler and Karadi \(2015\)](#). To compare our results to theirs, we replicate our analysis using the Romer and Romer shocks. We use the very same time series that they use in their baseline analysis, which they obtain by taking into account the nonlinear smooth transition of their model. As shown in [Appendix F](#) of the Online Appendix, this alternative specification suggests that the response of the interest rate reverts back to zero more rapidly in both recession and expansion compared to our specification, and that the effects on all remaining variables are usually stronger. However, the results are qualitatively very similar to our baseline result, and confirm the finding that in a recession, monetary policy has a stronger effect on financial variables, on inflation, and on output. The same figure shows that our results are also robust to using the instrument for monetary shocks by [Miranda-Agrippino and Ricco \(2021\)](#), although we stress that their instrument is the outcome of an estimation in a linear model.

4 Conclusions

While VAR models are identified using a wide range of identification strategies, Smooth Transition VAR models are typically identified using the recursive identification approach. This paper develops a tractable way of identifying Smooth Transition VAR models either via external instruments or via a combination of external instruments and sign restrictions. The Bayesian approach we pursue in this paper makes the implementation of proxies tractable despite the presence of non-linearities, offering a valid alternative to the recursive identification. It also simplifies inference on the key parameters ruling the nonlinearity of the model instead of calibrating them, because it uses a combination of model specification and prior distributions that allows for

the closed form derivations of the marginal posterior distribution of such parameters, hence avoiding the need for a Metropolis-Hastings-within-Gibbs sampler.

We apply the model to study if the effects of monetary policy shocks depend on the phase of the business cycle that prevails when the shock hits the economy. We build on their linear model by [Gertler and Karadi \(2015\)](#) and [Jarociński and Karadi \(2020\)](#) and extend it to interact it with the quarter-on-quarter growth in real GDP. The results indicate that the closer the economy is to a recession, the more the Spread by [Gilchrist and Zakrajšek \(2012\)](#) and the S&P500 respond to the monetary shock. This finding is consistent with our finding on output and inflation, which we document to be more responsive to a monetary shock in a recession than in an expansion.

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