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Abstract. A central assumption for identifying structural shocks in vector autoregressive (VAR) models via heteroskedasticity is the time-invariance of the impact effects of the shocks. It is shown how that assumption can be tested when long-run restrictions are available for identifying structural shocks. The importance of performing such tests is illustrated by investigating the impact of fundamental shocks on stock prices in the U.S.. It is found that fundamental shocks post-1986 have become more important than in the pre-1986 period.

Key Words: Structural vector autoregression, heteroskedasticity, cointegration, structural vector error correction model

JEL classification: C32

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1 Introduction

Identification of structural shocks through heteroskedasticity in structural vector autoregressive (VAR) analysis has been used extensively in recent years (see, e.g., Lütkepohl and Netšunajev (2017), Kilian and Lütkepohl (2017, Chapter 14), Lütkepohl and Velinov (2016)). A central assumption for using that device is that the impact effects of the shocks are time-invariant despite the change in the variance of the shocks. Some authors have pointed out that the assumption may be problematic (e.g., Angelini, Bacchiocchi, Caggiano and Fanelli (2019), Bacchiocchi, Castelnuovo and Fanelli (2018), Bacchiocchi and Fanelli (2015), Lütkepohl and Schlaak (2022), Bruns and Lütkepohl (2024)).

Another drawback of identification through heteroskedasticity is that the shocks identified in this way are purely statistical/mathematical shocks and may not correspond to economic shocks of interest. In any case, to interpret them as economic shocks additional subject matter or economic knowledge has to be available to label the shocks properly. Because of this obstacle of identification through heteroskedasticity the device has often been used only as an additional identification tool that leads to over-identified shocks and thereby enables the user to assess or compare different conventional identification assumptions (e.g., Lanne and Lütkepohl (2014), Lanne and Lütkepohl (2008), Lütkepohl and Netšunajev (2014), Lütkepohl and Velinov (2016), Chen and Netšunajev (2016), Netšunajev (2013)).

If the shocks can be identified via other means than heteroskedasticity and heteroskedasticity serves as an over-identifying device, then the assumption of time-invariance of the impact effects of the structural shocks can be tested. Lütkepohl and Schlaak (2022) and Bruns and Lütkepohl (2024) use this possibility for the case where proxy variables are available for identification. In this study, we will explore this option for structural VARs identified by long-run restrictions and discuss and investigate the properties of tests for time-varying impact effects in the context of VAR models identified by restrictions on the long-run effects of the shocks. As long-run restrictions for identifying structural shocks in VAR analysis are closely related to the cointegration structure of the underlying variables, we will consider a vector error correction model setup to discuss the issue (see, e.g., Granger (1986) for the concept of cointegration and early developments and King, Plosser, Stock and Watson (1991) for the relation between cointegration and structural VAR modelling).

We will use the tests to investigate the question whether U.S. stock prices are driven primarily by speculation or by fundamentals. The issue has been discussed extensively and with controversial opinions and results in the literature (see, e.g., Velinov (2013)). A number of studies have addressed the topic in the context of structural VAR models (e.g., Lee (1995), Rapach (2001), Binswanger (2004) and Jean and Eldomiaty (2010)). Lütkepohl and Velinov (2016) use identification through heteroskedasticity techniques to compare models with different sets of variables and different identification schemes for the fundamental shocks. We consider the time-invariance of the transmission of fundamental shocks at times of variance changes in a small benchmark model for the U.S. considered by Lütkepohl and Velinov (2016). It consists of three variables, real GDP (gdp_t), a real interest rate (r_t) and a real stock price index (s_t) and has been used to explore the impact of fundamental shocks on stock prices. Evidence is found that the importance of fundamental shocks has increased since the middle of the 1980s.

The remainder of the paper is organized as follows. The model setup and model estimation are presented in the next section. The tests for time-varying impact effects of the shocks and their small-sample properties are discussed in Section 3. The empirical study follows in Section 4 and conclusions are drawn in the final section. Some technical issues are deferred to the Appendix.

The following notation is used: Δ is the differencing operator such that $\Delta y_t = y_t - y_{t-1}$ for a time series or stochastic process y_t. The $(K \times K)$ identity matrix is denoted by I_K, while $0_{n \times m}$ is an $(n \times m)$ dimensional matrix of zeros. For a matrix A, A' is the transpose, A^{-1} denotes the inverse, A^+ the Moore-Penrose generalized inverse, and $\mathrm{rk}(A)$ signifies the rank. For a $(K \times N)$ matrix C, $N < K$, with $\mathrm{rk}(C) = N$, $C_\perp$ denotes a $(K \times (K-N))$ orthogonal complement of the matrix C such that $[C : C_\perp]$ is nonsingular and $C'C_\perp = 0$. If C is a nonsingular square matrix, $C_\perp = 0$ and if C is a zero matrix, $C_\perp$ is an identity matrix with suitable dimensions. The symbol vec denotes the column vectorizing operator such that, for a $(K \times N)$ matrix A, $\mathrm{vec}(A)$ is a KN-dimensional column vector and vech is the vectorization operator that collects the elements of a $(K \times K)$ square matrix from the main diagonal downward in a $K(K+1)/2$-dimensional column vector. D_K denotes the $(K^2 \times \frac{1}{2}K(K+1))$-dimensional duplication matrix defined such that, for a symmetric $(K \times K)$ matrix S, $\mathrm{vec}(S) = D_K \mathrm{vech}(S)$.

2 The Model

In this section we discuss the model setup and parameter estimation.

2.1 Model Setup

Our point of departure is a standard K-dimensional reduced-form VAR(p) model for a vector of time series variables $y_t = (y_{1t}, \ldots, y_{Kt})'$,

$$y_t = A_1 y_{t-1} + \cdots + A_p y_{t-p} + u_t, \tag{1}$$

where A_j $(j = 1, \dots, p)$ are $(K \times K)$ VAR slope coefficient matrices and u_t is a zero-mean white noise error term with nonsingular, possibly time-varying covariance matrix Σ_t. In other words, $u_t \sim (0, \Sigma_t)$. Thus, we allow explicitly for heteroskedasticity. Note that there is no deterministic term in the model (1) because such terms do not play a role for structural analysis in our framework. Of course, in practice, deterministic terms have to be added for an empirical analysis, as required by the data features.

For the residual covariance matrices we consider a simple model with a finite number of volatility regimes as in Lütkepohl and Schlaak (2022) and Bruns and Lütkepohl (2024). Specifically, the residual covariances are

$$\mathbb{E}(u_t u_t') = \Sigma_t = \Sigma_u(m) \quad \text{for} \quad t \in \mathcal{T}_m, \quad m = 1, \dots, M, \tag{2}$$

where $\mathcal{T}_m = \{T_{m-1} + 1, \dots, T_m\}$ $(m = 1, \dots, M)$ are M volatility regimes with volatility change points at the end of time periods T_m, for $m = 1, \dots, M - 1$ and we specify $T_0 = 0$ and $T_M = T$, the overall sample size.

For our analysis we assume that all components of y_t are stationary $(I(0))$ or integrated of order one $(I(1))$. Extensions for higher-order integrated variables are possible but are not needed for our empirical analysis and would complicate the notation and distract from the main points of the paper. To focus the analysis of the cointegration properties of the variables and assuming that there are r $(0 \leq r \leq K)$ linearly independent cointegration relations between the components of y_t, we convert the model (1) to a vector error correction model (VECM) as proposed by Johansen and Juselius (1990) and Johansen (1991, 1995),

$$\Delta y_t = \alpha\beta' y_{t-1} + \Gamma_1 \Delta y_{t-1} + \cdots + \Gamma_{p-1} \Delta y_{t-p+1} + u_t, \tag{3}$$

where α is a $(K \times r)$ loading matrix of rank r, β is a $(K \times r)$ cointegration matrix, also of rank r, and $\Gamma_1, \dots, \Gamma_{p-1}$ are $(K \times K)$ slope coefficient matrices (see, e.g., Lütkepohl (2005) for the relation between the parameters in (1) and (3)). For $r = 0$, the error correction term $\alpha\beta' y_{t-1} = 0$ and, hence, disappears in (3) and the model is a VAR$(p - 1)$ in first differences of the variables. On the other side, if $r = K$, $\alpha\beta' = \Pi = -(I_K - A_1 - \cdots - A_p)$ is an invertible $(K \times K)$ matrix. Identification of the structural shocks in this framework was proposed by King et al. (1991).

The structural shocks, w_t, are typically obtained by a linear transformation from the reduced-form residuals, i.e., $w_t = B^{-1} u_t$ such that the components of w_t are instantaneously uncorrelated. In other words, $\mathbb{E}(w_t w_t') = B^{-1} \Sigma_t B^{-1\prime}$ is a potentially time-varying diagonal matrix. Given our assumption for the heteroskedasticity of the reduced-form residuals in (2), for $t \in \mathcal{T}_m$ the variances of the structural shocks may depend on the volatility regime $\mathcal{T}_m$ and we denote the covariance matrix of w_t for $t \in \mathcal{T}_m$ by $\Sigma_w(m)$.

Substituting Bw_t for u_t in (1) or (3), the matrix B is easily recognized as the matrix of impact effects of the structural shocks. Heteroskedasticity has been used in structural VAR analysis to identify B. However, this identification device relies on B being time-invariant. Clearly, if the variances of the structural shocks change, it is possible that also their impact on the variables changes and, hence, their transmission through the system. Consequently, we also allow the impact effects of the shocks to depend on the volatility regime and denote the matrix of impact effects associated with volatility regime m by $B(m)$. In the following we will present a test for time-varying impact effects that checks whether elements of $B(m)$ are equal to the corresponding elements of $B(n)$ for $m \neq n$.

The long-run effects of the structural shocks with impact effects $B(m)$ are known to be

$$\Upsilon(m) = \Xi B(m), \tag{4}$$

where

$$\Xi = \beta_\perp \left[\alpha'_\perp \left(I_K - \sum_{i=1}^{p-1} \Gamma_i \right) \beta_\perp \right]^{-1} \alpha'_\perp \tag{5}$$

is a matrix of rank $K - r$ (see, e.g., Lütkepohl (2005, Chapter 9) or Kilian and Lütkepohl (2017, Chapter 10) for details). Obviously, the long-run effects of the shocks may also depend on the volatility regime m if the impact effects may depend on m. As $B(m)$ is a full rank matrix, $\Upsilon(m)$ inherits the rank of Ξ and, for $r < K$, also has rank $K-r$. If $r = 0$ and, hence, the VAR process (1) is a stable VAR($p-1$) in the first differences of all variables, then Ξ becomes

$$\Xi = \left(I_K - \sum_{i=1}^{p-1} \Gamma_i \right)^{-1}. \tag{6}$$

In that case, $\Upsilon(m)$ also has rank K and represents the matrix of cumulated effects of the shocks. In other words, for integrated variables that appear in first differences in y_t, $\Upsilon(m)$ contains the effects on the related levels variables.

In any case, as the reduced-form of the model represents the DGP and is, hence, given for a specific data set, imposing restrictions on the long-run effects $\Upsilon(m)$, implies restrictions for the impact effects $B(m)$.

The reduced rank of $\Upsilon(m)$ implies that there can only be as many shocks without any long-run effects, corresponding to columns of zeros in $\Upsilon(m)$, as there are cointegration relations. In this sense, the number of cointegration relations is central for the potential properties of structural VAR shocks. If the cointegration rank is zero, all shocks must have permanent effects at least on some of the variables. Since the rank of $\Upsilon(m)$ is $(K-r)$, it is in general not sufficient to identify all shocks by $K(K-1)/2$ restrictions on the matrix of long-run effects.

There is a large body of literature on long-run restrictions for identifying structural shocks in VARs. For example, there are proposals by Blanchard and Quah (1989), Gonzalo and Ng (2001), Fisher, Huh and Summers (2000), and Pagan and Pesaran (2008). Fisher and Huh (2014) review that literature and discuss the relations between the various approaches. In the present context it is not important which approach is used for imposing long-run restrictions. They can all be combined with the identification procedures derived from time-varying volatility, as discussed in the following.

2.2 Estimation

Estimation of the reduced-form VECM (3) is conveniently done by the Johansen (1991, 1995) Gaussian maximum likelihood (ML) procedure, ignoring the heteroskedasticity in a first step. It provides good asymptotic properties of the estimators for a large family of distributions and can be applied even if the actual distribution of the data is non-Gaussian. We denote the resulting estimates by $\hat{\alpha}, \hat{\beta}, \hat{\Gamma}_1, \dots, \hat{\Gamma}_{p-1}$ and $\widehat{\Sigma}_u(m)$, $m = 1, \dots, M$, where the latter estimates are obtained from the VECM residuals $\hat{u}_t$ as

$$\widehat{\Sigma}_u(m) = \frac{1}{T_m - T_{m-1}} \sum_{t \in \mathcal{T}} \hat{u}_t \hat{u}_t'.$$

Choosing any orthogonal complements $\hat{\alpha}_\perp$ and $\hat{\beta}_\perp$ of $\hat{\alpha}$ and $\hat{\beta}$, respectively, we can estimate Ξ as

$$\widehat{\Xi} = \hat{\beta}_\perp \left[\hat{\alpha}_\perp' \left(I_K - \sum_{i=1}^{p-1} \hat{\Gamma}_i \right) \hat{\beta}_\perp \right]^{-1} \hat{\alpha}_\perp'$$

If enough restrictions for the $B(m)$ and the diagonal elements of the $\Sigma_w(m)$ are available such that they are just-identified, then these parameters can be estimated by solving the set of equations

$$\operatorname{vech}\left(B(m)\Sigma_w(m)B(m)' = \widehat{\Sigma}_u(m) \right)$$

subject to all restrictions on $B(m)$ and $\widehat{\Xi}B(m)$, for $m = 1, \dots, M$, and a diagonal $\Sigma_w(m)$ matrix. The computations may be easier if we consider an impact effects matrix $B^*(m) = B(m)\Sigma_w^{1/2}$ such that $B^*(m)B^*(m)' = \Sigma_u(m)$ and we can solve for $B^*(m)$ satisfying

$$\operatorname{vech}\left(B^*(m)B^*(m)' = \widehat{\Sigma}_u(m) \right). \tag{7}$$

Once we have found the estimate $\hat{B}^*(m)$ we can then standardize one element in each column to be unity. In other words, we divide each column by one nonzero element. Thereby we assign a unit response on impact to one of the variables. Thus, for each shock, a variable has to be chosen that has a nonzero response on impact. Of course, the same variable should be used in each volatility regime, i.e., for $m = 1, \dots, M$.

Interpreting $\widehat{\Sigma}_u(m)$ as a GMM estimator, we get from general GMM results that, under general conditions, the estimator is consistent and asymptotically normal if we assume that the regimes get larger with the sample size. We are assuming in the following that the fractions $\tau_m = (T_m - T_{m-1})/T$ are fixed. By Slutsky's Theorem, the resulting estimators of $B(m)$ and $B^*(m)$ will then also be asymptotically normal. Given the identifying restrictions on these matrices, they will have singular asymptotic distributions which has to be taken into account in testing for time-variation across volatility regimes, as discussed in Section 3.

Because we want to accommodate changing variances of the structural shocks across the volatility regimes even if the impact effects do not change, we have to disentangle the contribution of shock variance changes and changes in the impact effects in the $B^*(m)$ matrices. To ensure comparable $B(m)$ matrices across volatility states, we normalize the impact effects of the shocks such that each shock has a unit impact effect on one of the variables across all volatility states. In other words, we restrict one element in each of the columns of the $B(m)$ matrices to be 1 for all $m = 1, \dots, M$. To obtain estimators of the $B(m)$ matrices, we use the $\hat{B}^*(m)$ matrices and divide each column by the estimate corresponding to the unit element in the corresponding column of $B(m)$. The estimator will be denoted by $\hat{B}(m)$ in the following. Clearly, for the normalisation of the columns of $B(m)$, the elements restricted to 1 have to be well away from zero. In other words, we have to choose variables which clearly respond to the shock of interest on impact.

3 Testing for Time-Varying Impact Effects

3.1 The Tests

Given that the asymptotic distribution of the estimated impact effects matrix $B(m)$ is singular, special care is required in setting up tests for hypotheses regarding the elements of $B(m)$. Note that some elements of $B(m)$ are 1 and there may also be elements that are restricted to zero. In any case the identifying restrictions are imposed on its elements. Therefore, testing the pair of hypotheses

$$\mathbb{H}_0 : B(m) = B(n) \quad \text{vs.} \quad \mathbb{H}_1 : B(m) \neq B(n) \tag{8}$$

is not straightforward in general and requires careful examination of the identifying restrictions imposed on the elements of the matrices involved.

In general, it will be easy, however, to set up corresponding tests for individual elements of the $B(m)$ matrices or columns of the matrices. For the k-th column, say $b_k(m)$ of $B(m)$, a pair of hypotheses of specific interest is

$$\mathbb{H}_0 : b_k(m) = b_k(n) \quad \text{vs.} \quad \mathbb{H}_1 : b_k(m) \neq b_k(n). \tag{9}$$

Under our assumptions, the impulse responses of the k-th shock will be identical in volatility regimes m and n if the null hypothesis holds. In that case, if the shock variances in regimes m and n are different, this does not affect the responses of the variables to the shock.

For pairs of hypotheses such as (9) it may be easier to determine the asymptotic distribution of the unrestricted elements and use that for setting up a standard Wald type test. Suppose, for example, the only fully restricted element in the k-th column is the element normalized to 1, then, denoting the $(K-1)$-dimensional vector of unrestricted elements of $b_k(m)$ by $\eta_k(m)$, we have

$$\sqrt{T}(\hat{\eta}_k(m) - \eta_k(m)) \xrightarrow{d} \mathcal{N}(0, \tau_m^{-1}\Sigma_{\eta_k}(m)), \tag{10}$$

where $\hat{\eta}_k(m)$ is the estimator implied by $\hat{b}_k(m)$.

The covariance matrix $\Sigma_{\eta_k}(m)$ will typically be nonsingular such that we can use a Wald statistic

$$W(\eta_k) = T(\hat{\eta}_k(m) - \hat{\eta}_k(n))' \left(\tau_m^{-1}\widehat{\Sigma}_{\eta_k}(m) + \tau_n^{-1}\widehat{\Sigma}_{\eta_k}(n)\right)^{-1} (\hat{\eta}_k(m) - \hat{\eta}_k(n)) \tag{11}$$

with a $\chi^2(K-1)$ distribution to test the pair of hypotheses

$$\mathbb{H}_0 : \eta_k(m) = \eta_k(n) \quad \text{vs.} \quad \mathbb{H}_1 : \eta_k(m) \neq \eta_k(n) \tag{12}$$

which in this case is equivalent to the pair of hypotheses in (9). Note that $\widehat{\Sigma}_{\eta_k}(m)$ denotes a consistent estimator of $\Sigma_{\eta_k}(m)$ and we assume that the residual process u_t is such that $\hat{\eta}_k(m)$ and $\hat{\eta}_k(n)$ are asymptotically independent and, hence, the asymptotic covariance of $\sqrt{T}\left((\hat{\eta}_k(m) - \eta_k(m)) - (\hat{\eta}_k(n) - \eta_k(n))\right)$ is $(\tau_m^{-1}\Sigma_{\eta_k}(m) + \tau_n^{-1}\Sigma_{\eta_k}(n))$. In the Monte Carlo simulations in Section 3.2 and in the empirical application in Section 4, we will estimate the covariance matrix used for the Wald statistic by a bootstrap procedure (see Appendix A.2.1 for details).

3.2 Small Sample Properties

We investigate the small sample properties of the test using a Monte Carlo experiment based on two different data generating processes (DGPs). The first one (DGP1) has cointegration rank $r = 0$ while the cointegration rank of the second one (DGP2) is $r = 1$.

3.2.1 DGP1

The parameters of DGP1 are informed by the empirical Model I in Lütkepohl and Velinov (2016) using the updated series also employed in Section 4. We use a VAR(2) model in first differences of the variables with constant term and the VAR slope coefficients from the application in Section 4. This yields:

$$\Xi = \begin{bmatrix} 1.496 & -0.001 & 0.082 \\ 23.937 & 0.634 & 2.820 \\ -0.796 & -0.006 & 1.047 \end{bmatrix}.$$

We also set

$$B(1) = \begin{bmatrix} 1.00 & 0.00 & -0.06 \\ 33.24 & 1.00 & -2.31 \\ 12.35 & 0.02 & 1.00 \end{bmatrix}$$

corresponding to the impact effects matrix in the second volatility regime in our application in Section 4 such that $\Xi B(1)$ is a lower-triangular matrix, and

$$B(2) = \begin{bmatrix} 1.00 & 0.00 & -0.06 \\ \delta \times 33.24 & 1.00 & -2.31 \\ \delta \times 12.35 & 0.02 & 1.00 \end{bmatrix},$$

where δ is a scalar that varies in the power analysis of the tests. The structural error covariance matrices for the two regimes are $\Lambda(1) = \text{diag}(0.0005, 0.0154, 0.3113)$ and $\Lambda(2) = \text{diag}(0.0001, 0.0017, 0.0195)$.

To generate samples of y_t we draw reduced-form errors $u_t \sim \mathcal{N}(0, B(1)\Lambda(1)B(1)')$ for the first volatility regime and $u_t \sim \mathcal{N}(0, B(2)\Lambda(2)B(2)')$ for the second. Under $\mathbb{H}_0$ we set $\delta = 1$ and under $\mathbb{H}_1$ we use $\delta = 1.2$ and 1.5.

We generate 500 samples of size $T = 150, 250$, and 500 with a breakpoint $T_1 = T/2$ and fit a VAR with the same specifications as the DGP, i.e., the lag order is 2 and it has a constant term. The covariance estimator in the Wald statistic (11) is based on 500 bootstrap replications. Table 1 reports the rejection frequencies for testing time-invariance of the impact effects of the first shock ($\mathbb{H}_0 : \eta_1(1) = \eta_1(2)$ vs. $\mathbb{H}_1 : \eta_1(1) \neq \eta_1(2)$) when data are generated under $\mathbb{H}_0$ and $\mathbb{H}_1$ using nominal levels of 5% and 10%.

The relative rejection frequencies under $\mathbb{H}_0$ in Table 1 are reasonably close to the nominal levels even for relatively small samples of size $T = 150$, while the finite sample power is quite substantial for all scenarios considered. The power increases with the sample size and with the distance of the alternative from $\mathbb{H}_0$ measured by δ, as one would expect. In other words, a larger δ leads to higher small sample power if there is still room for an increase in the rejection frequency,

Table 1: Relative Rejection Frequencies (DGP1)

	Nominal level 5%			Nominal level 10%		
		$\mathbb{H}_1$			$\mathbb{H}_1$	
	$\mathbb{H}_0$	$\delta = 1.2$	$\delta = 1.5$	$\mathbb{H}_0$	$\delta = 1.2$	$\delta = 1.5$
$T = 150$	0.062	0.898	0.954	0.090	0.904	0.956
$T = 250$	0.042	0.984	0.996	0.084	0.986	0.998
$T = 500$	0.038	1.000	1.000	0.070	1.000	1.000

that is, if the rejection frequency is not one already for the smaller δ. Overall, for DGP1, our test performs quite well in samples of a size which is often encountered in empirical macroeconometric studies.

3.2.2 DGP2

The parameters of DGP2 are informed by the empirical model referred to as Model II in Lütkepohl and Velinov (2016). This model uses $K = 3$ U.S. variables, namely log real earnings, real interest rates, and log real stock prices, all deflated using the CPI inflation rate. Quarterly data from 1947Q1 - 2007Q3 are used, implying $T = 240$ observations. Lütkepohl and Velinov (2016) use data until 2012Q1, while we discard the last 5 years to avoid the volatility cluster during the financial crisis in 2007/08. Following Lütkepohl and Velinov (2016), and supported by the test in Cavaliere, Angelis, Rahbek and Taylor (2018), we estimate a VECM as in (3) with a constant term, cointegration rank $r = 1$ and lag length $p = 3$. This yields:

$$\nu = \begin{bmatrix} -0.012 \\ 0.710 \\ 0.030 \end{bmatrix}, \quad \alpha = \begin{bmatrix} 0.020 \\ -0.107 \\ -0.036 \end{bmatrix}, \quad \beta = \begin{bmatrix} 1 \\ -0.063 \\ -0.521 \end{bmatrix},$$

and

$$\Gamma = \begin{bmatrix} 1.112 & 0.003 & 0.031 & -0.378 & 0.001 & 0.006 \\ 6.720 & -0.033 & -0.485 & -3.896 & -0.281 & 0.901 \\ 0.078 & -0.011 & 0.322 & -0.132 & -0.007 & -0.018 \end{bmatrix}.$$

We assume a volatility breakpoint in 1986Q3, as for the somewhat different empirical model considered in Section 4, and obtain the regime-dependent impact effects imposing the restrictions

$$\Xi B(m) = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & 0 \end{bmatrix}$$

as in Lütkepohl and Velinov (2016) by minimising vech$[B^*(m)B^*(m)' - \widehat{\Sigma}_u(m)]$. After standardising the (2,1) element of $B(m)$ this yields for $\delta = 1$:

$$B(1) = \begin{bmatrix} 0.014 & 0.003 & -0.010 \\ 1 & 0.560 & 0.552 \\ 0.031 & -0.038 & 0.018 \end{bmatrix}, \quad B(2) = \begin{bmatrix} \delta \times 0.037 & 0.009 & -0.008 \\ 1 & 0.190 & 0.460 \\ \delta \times 0.025 & -0.048 & 0.015 \end{bmatrix}.$$

As for DGP1, we vary the scalar δ and thereby the parameters considered under $\mathbb{H}_1$ for the power analysis of our test.

To generate data under $\mathbb{H}_0$, we set $B = B(1)$ and generate reduced-form errors $u_t \sim \mathcal{N}(0, BB')$ for the first volatility regime and $u_t \sim \mathcal{N}(0, B\Lambda B')$ for the second regime, where $\Lambda = \text{diag}(0.690, 0.387, 0.330)$.[2] To generate data under $\mathbb{H}_1$ we generate reduced-form errors $u_t \sim \mathcal{N}(0, B(1)B(1)')$ for the first volatility regime and $u_t \sim \mathcal{N}(0, B(2)B(2)')$ for the second regime.

We generate 500 samples of size $T = 150, 250$, and 500 with a breakpoint $T_1 = T/2$ and fit a VECM with the same specifications as the DGP, i.e., $r = 1$ and $p = 3$. We consider two values of δ ($\delta = 0.5$ and 1). Both values make the first column of $B(2)$ clearly distinct from the first column of $B(1)$. Although all columns of $B(1)$ and $B(2)$ differ in this simulation design, we test again time-invariance of the first column only ($\mathbb{H}_0 : \eta_1(1) = \eta_1(2)$ vs. $\mathbb{H}_1 : \eta_1(1) \neq \eta_1(2)$). As for DGP1, 500 bootstrap replications are used to compute the covariance estimator for the test statistic in (11).

Table 2: Relative Rejection Frequencies (DGP2)

	Nominal level 5%			Nominal level 10%		
		$\mathbb{H}_1$			$\mathbb{H}_1$	
	$\mathbb{H}_0$	$\delta = 0.5$	$\delta = 1$	$\mathbb{H}_0$	$\delta = 0.5$	$\delta = 1$
$T = 150$	0.028	0.073	0.800	0.036	0.117	0.853
$T = 250$	0.022	0.498	1.000	0.031	0.619	1.000
$T = 500$	0.040	0.990	1.000	0.061	0.994	1.000

Table 2 reports the relative rejection frequencies for DGP2 when data are generated under $\mathbb{H}_0$ and $\mathbb{H}_1$ using nominal significance levels of 5% and 10%, respectively. For DGP2 the test appears to be slightly undersized for small samples with relative rejection frequencies below 5% and 10%, respectively, for sample sizes up to $T = 500$. Despite this feature, the power is remarkable and grows quickly

[2]The elements of Λ are obtained as $\underset{\Lambda}{argmin}\ \text{vec}(B\Lambda B' - \Sigma(2))'\text{vec}(B\Lambda B' - \Sigma(2))$ and $\Sigma(2) = B(2)B(2)'$. In other words, to generate data under $\mathbb{H}_0$, we scale the variance of the structural shocks in the second volatility regime to approximately match the reduced-form covariance matrix in the second regime when data are generated under $\mathbb{H}_1$.

with the sample size. For example, for a significance level of 5%, the power for $\delta = 0.5$ is rather small (0.073) for a sample size of $T = 150$ but it is already almost 0.5 for $T = 250$ and close to one for $T = 500$. Thus, overall the test performs quite well in small samples also for our DGP2 which has cointegration rank 1. Generally, the simulation results for DGP1 and DGP2 are a strong encouragement for using the test in practice and we will do so in the next section.

4 Empirical Investigation of Stock Price Fundamentals

To consider the question to what extent stock prices reflect their underlying economic fundamentals or are primarily driven by speculation, we perform an empirical analysis using a benchmark model from Velinov (2013) and Lütkepohl and Velinov (2016). The latter authors condition on time-invariant impact effects of their structural shocks and use the implied over-identifying restrictions due to heteroskedasticity to test identifying long-run restrictions. Instead, we will use the over-identifying information from the long-run restrictions and heteroskedasticity to test the time-invariance of the impact effects. Thereby we will also demonstrate the usefulness of tests for time-varying impact effects of structural shocks at times of volatility changes.

The dividend discount model (DDM) states that the price of an asset is the sum of its expected future discounted payoffs. As the payoffs are linked to real economic activity such as real GDP, stock prices should also depend on real economic factors. Alternatively, stock prices may be driven primarily by speculation and, hence, may depend on economic fundamentals only to a limited extent or not at all. The issue has been considered by a number of authors (see, e.g., Lee (1995), Rapach (2001), Binswanger (2004), Lanne and Lütkepohl (2010) and Jean and Eldomiaty (2010)).

We consider a model for the U.S. consisting of the three quarterly variables real GDP (gdp_t), real interest rates (r_t), and real stock prices (s_t). In other words, $y_t = (gdp_t, r_t, s_t)'$. Related models have been used in the aforementioned studies by other authors. Our sample period extends from 1947Q1 to 2019Q4 giving a gross sample size of 292 observations. We do not include the COVID period to avoid having to deal with possibly more substantial structural changes. Lütkepohl and Velinov (2016) use a shorter sample period from 1947Q1 - 2012Q3. More details on our data and their sources are provided in Appendix A.1.

To set up the reduced-form VECM, we first investigate the lag order and cointegration rank. Based on standard unit root and cointegration tests, for their dataset, Lütkepohl and Velinov (2016) consider a VAR(2) model in first differences which corresponds to a cointegration rank zero. As there may be heteroskedastic-

Table 3: BIC Values for Lag Order, p, and Cointegration Rank, r

	lag order p							
r	1	2	3	4	5	6	7	8
0	-14.715	-14.837	**-14.843**	-14.745	-14.575	-14.464	-14.330	-14.202
1	-14.662	-14.772	-14.772	-14.672	-14.503	-14.402	-14.276	-14.137
2	-14.641	-14.731	-14.716	-14.622	-14.453	-14.361	-14.236	-14.096
3	-14.636	-14.713	-14.693	-14.606	-14.441	-14.346	-14.223	-14.079

ity in the residuals, we use a heteroskedasticity-robust procedure for determining the VAR lag order and the cointegration rank jointly, as proposed by Cavaliere et al. (2018). It is based on minimizing the BIC criterion across a range of pairs of lag orders and cointegration ranks. The details are given in Appendix A.2.2. We include a deterministic trend in the levels version of our VAR model.

The results are presented in Table 3. The BIC criterion is minimized for lag order $p = 3$ and cointegration rank $r = 0$. We proceed with this lag order and cointegration rank and, accordingly, we use a VAR(2) model with constant intercept term in first differences of all three variables which corresponds to $p = 3$ for the model in levels. Thus, for our longer sample, using the same settings as in Lütkepohl and Velinov (2016) is suggested by our statistical procedure.

The VAR(2) residuals for the first differences of the variables are plotted in Figure 1. They show clear signs of heteroskedasticity. Therefore we search for a single volatility break point by minimizing the criterion function

$$\psi(T_1) = T_1 \log\det\widehat{\Sigma}_u(1) + (T - T_1)\log\det\widehat{\Sigma}_u(2) \tag{13}$$

over $T_1 \in \{0.15T, \dots, 0.85T\}$. Thereby we find a volatility change point in 1986Q3. Conditioning on this change point and searching for an additional volatility change point results in 1978Q2. These change points roughly coincide with the period when Volcker was the Fed Chairman which is known as a period of high macroeconomic volatility in the U.S.. In the following we will investigate time-invariance of the impact effects of the shocks in a model with $M = 2$ volatility regimes and a volatility change point in 1986Q3 as well as in a model with $M = 3$ volatility regimes specified by change points 1978Q2 and 1986Q3.

We have used an LM test for heteroskedasticity as described in Lütkepohl (2005, pp. 600-601) to investigate whether the residual covariances in all three potential volatility regimes are in fact distinct. The tests yield very small p-values below 0.1% and thereby support the notion of different variances in the three volatility regimes (see Table 4).

The structural parameters are identified by restrictions on the long-run effects. Only the first shock is allowed to have permanent effects on all the variables of

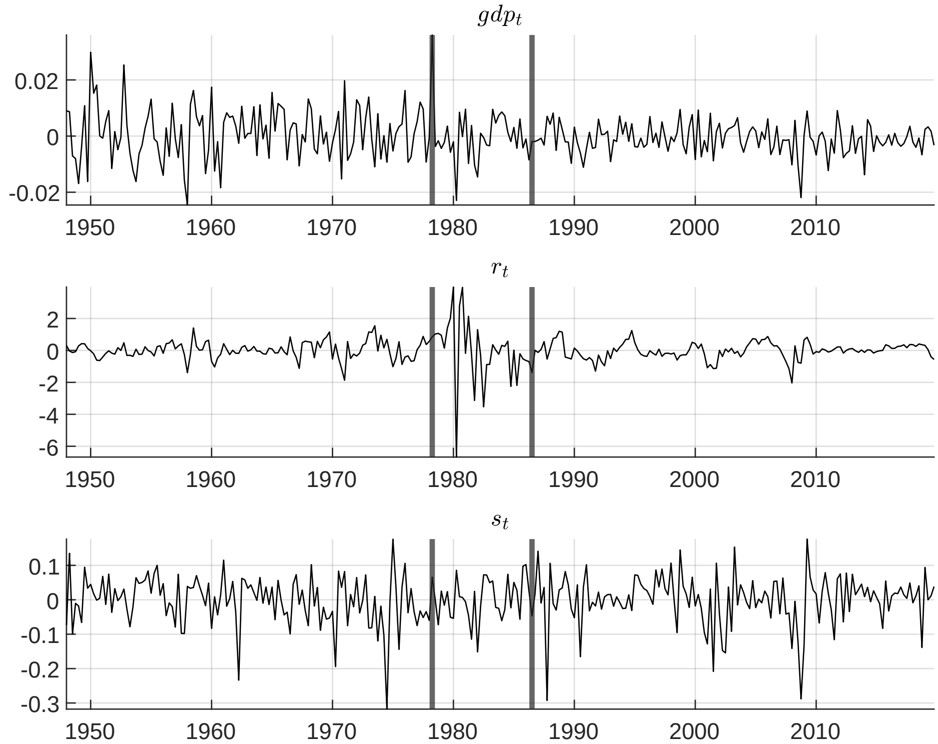

Figure 1: Residuals of VAR(2) for first differences of variables.

the system and is therefore labelled as fundamental. Lütkepohl and Velinov (2016) consider a time-invariant impact effects matrix B and identify the structural shocks by standardizing their variances to unity and by choosing a lower-triangular long-run effects matrix

$$\Upsilon = \Xi B = \begin{bmatrix} * & 0 & 0 \\ * & * & 0 \\ * & * & * \end{bmatrix}, \tag{14}$$

where $*$ signifies an unrestricted element. These restrictions just-identify the structural parameters. We use the same lower-triangular structure for each of the M long-run effects matrices, that is, we choose all $\Upsilon(m)$, $m = 1, \dots, M$, to be lower-triangular. In this case, taking into account the cointegration rank of zero ($r = 0$), estimation of $B(m)$ is particularly simple because a closed-form estimator for $B^*(m)$ exists (see Appendix A.2.3).

To allow for time-varying variances of the structural shocks, we restrict the diagonal elements of each $B(m)$ matrix to be unity. In other words,

$$B(m) = \begin{bmatrix} 1 & * & * \\ * & 1 & * \\ * & * & 1 \end{bmatrix}. \tag{15}$$

Table 4: Tests for Heteroskedasticity for Reduced-Form VAR Model

Period 1	versus	Period 2	Test statistic	p-value
1947Q1-1978Q2	versus	1978Q3-1986Q3	162.61	$< 0.1\%$
1947Q1-1978Q2	versus	1986Q4-2019Q4	67.65	$< 0.1\%$
1978Q3-1986Q3	versus	1986Q4-2019Q4	211.32	$< 0.1\%$
1947Q1-1986Q3	versus	1986Q4-2019Q4	105.73	$< 0.1\%$

Note: The LM test for heteroskedasticity described in Lütkepohl (2005, pp. 600-601) was used to test whether the residual covariance in period 1 is different from the residual covariance in period 2.

For the present analysis, the crucial shock of interest is the first shock, the fundamental shock, and that shock is clearly expected to have a nonzero impact effect on the first variable, GDP growth. Thus, standardizing the upper left-hand element of the impact effects matrix to one is justified, while it is of limited importance where the ones are placed in the other two columns. It is essential, of course, that the corresponding coefficients of the $B^*(m)$ matrices are nonzero.

As the cointegration rank is zero, the tests for time-varying impact effects of the individual shocks are very easy to set up because the two unrestricted elements in the first column of each of the $B(m)$ matrices are easily seen to have a nonsingular asymptotic distribution such that we can use the Wald statistic with a $\chi^2(2)$ distribution to test hypotheses of the type shown in (12). Note, however, that our setup implies that the last column of $B(m)$ is time-invariant by construction. It depends on the VECM slope coefficients only which are assumed to be time-invariant.[3] Thus, no tests are needed to verify time-invariant impact effects of the third shock, corresponding to the third column of $B(m)$. Test results for the first two columns are presented in Table 5.

If we consider just one volatility change ($M = 2$) in period 1986Q3, the Wald test results in a p-value of 0.000 and, hence, clearly rejects time-invariance of the impact effects of the fundamental shock at any reasonable significance level. On the other hand, time-invariance of the impact effects of the second shock cannot be rejected, given the p-value 0.334. Considering three volatility regimes, it turns out that there is weak evidence at best of a change in the impact effects in Regimes 1 and 2, while the impact effects of the fundamental shock in Regimes 1 and 2 differ significantly from those in Regime 3. Note that testing $\mathbb{H}_0 : \eta_1(1) = \eta_1(2)$ results in a p-value of 0.060 such that the test does not reject at significance level of 5%. Again no significant changes in the impact effects of the second shock across the sample are diagnosed. Thus, we find a clear change in the impact effects of the

[3] The last column of the matrix $B^*(m)$ is a scalar multiple of a column of the matrix Ξ^{-1} that depends on time-invariant parameters only. The scalar cancels in the transition to $B(m)$.

Table 5: Tests for Time-Varying Impact Effects

	$\mathbb{H}_0$	Wald statistic	p-value
$M = 2$	$\eta_1(1) = \eta_1(2)$	12.47	0.002
	$\eta_2(1) = \eta_2(2)$	2.46	0.292
$M = 3$	$\eta_1(1) = \eta_1(2)$	5.61	0.060
	$\eta_1(1) = \eta_1(3)$	15.24	0.000
	$\eta_1(2) = \eta_1(3)$	13.82	0.001
	$\eta_2(1) = \eta_2(2)$	0.12	0.940
	$\eta_2(1) = \eta_2(3)$	2.04	0.361
	$\eta_2(2) = \eta_2(3)$	3.36	0.187

Note: For $M = 2$ volatility regimes the change point is 1986Q3 while for $M = 3$ the change points are 1978Q2 and 1986Q3.

fundamental shock roughly at the time when the Great Moderation started,[4] while the possible volatility change in 1978Q2 has not resulted in clear changes in the impact effects. Therefore we continue under the assumption of a change in the impact effects of the fundamental shock in 1986Q3. Of course, we will account in the following analysis for possible heteroskedasticity also during the pre-1986Q3 period.

Given our test results, one would expect the impulse responses of the fundamental shock pre- and post-1986Q3 to be different. An important question in this context is, of course, the responses of which of the variables have changed and, importantly, whether the responses of the stock prices have changed. Therefore we present the impulse responses of the fundamental shock in Figure 2. Clearly, the responses of the stock index are much stronger in the second part of the sample (post-1986Q3). The same is also true for GDP and the interest rate. Thus, fundamental factors seem to have become more important, a result that is covered up if one conditions on time-invariant shock transmission as in Lütkepohl and Velinov (2016). Note, however, that the latter authors used a different model for volatility changes such that their analysis is not directly comparable to ours.

To further investigate the importance of fundamental shocks for the stock market, we report forecast error variance decompositions in Table 6. If one considers a time-invariant forecast error variance decomposition for the full sample period, fundamental shocks only contribute around 33% for short horizons and 26% after five years to the forecast error variance of the stock index. The situation changes substantially if we allow for a change in the shock transmission in 1986Q3. In that

[4]The exact timing of the beginning of the Great Moderation is controversial in the related literature (see, e.g., Stock and Watson (2003), Bernanke and Mihov (1998), McConnell and Perez-Quiros (2000), Galí and Gambetti (2009)). There seems to be consesus that it has started somewhere in the middle of the 1980s.

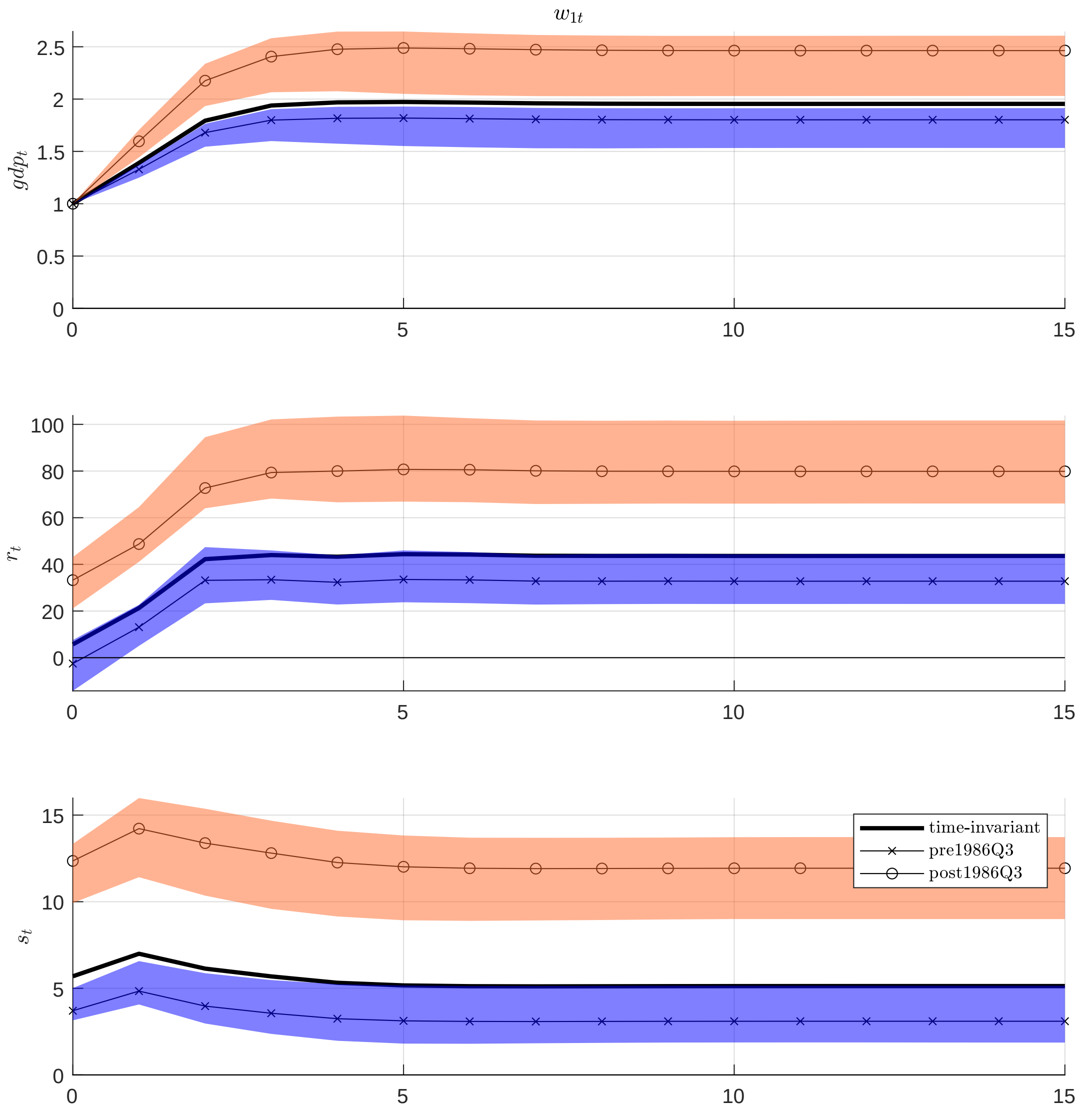

Figure 2: Responses to fundamental shocks. The coloured areas are 68% pointwise confidence intervals based on a moving block bootstrap implemented as in the Online Supplement of Bruns and Lütkepohl (2022). This procedure is asymptotically valid in conditionally heteroskedastic structural VAR models (see Jentsch and Lunsford (2019)) and allows for the possibility of further volatility changes within the regimes.

Table 6: Forecast Error Variance Decompositions for the S&P500 Stock Index

	full sample			pre-1986Q3			post-1986Q3		
forecast horizon	fund. shock	2nd shock	3rd shock	fund. shock	2nd shock	3rd shock	fund. shock	2nd shock	3rd shock
1	0.331	0.000	0.669	0.234	0.015	0.752	0.571	0.021	0.408
2	0.358	0.003	0.638	0.266	0.034	0.701	0.586	0.014	0.400
3	0.338	0.004	0.658	0.243	0.040	0.717	0.573	0.013	0.415
4	0.321	0.005	0.674	0.224	0.043	0.733	0.561	0.012	0.426
8	0.286	0.005	0.709	0.186	0.046	0.768	0.537	0.012	0.451
12	0.273	0.005	0.722	0.172	0.047	0.781	0.528	0.012	0.459
20	0.263	0.005	0.733	0.161	0.048	0.791	0.521	0.013	0.467

Note: Some shares do not sum to one due to rounding errors.

case, the fundamental shock has a substantially larger contribution to the forecast error variance post-1986Q3 than pre-1986Q3. For example, post-1986Q3, 57% of the forecast error variance for a one-quarter ahead forecast are due to fundamental shocks, while only 23% are explained by fundamental shocks in the pre-1986Q3 period. Although the share of fundamental shocks declines slightly for longer forecast horizons, it remains substantially higher in the post-1986Q3 period. Correspondingly, the contribution of the other shocks, capturing possibly speculation, declines post-1986Q3. This pattern is stable across the forecast horizons shown in Table 6.

Overall our results clearly indicate that stock prices are driven much more by fundamental shocks in the latter part of our sample than in the earlier sample period. Correspondingly, the importance of speculation has declined during the great moderation period relative to its importance in the earlier period. Such a finding is covered up if the impact effects of the shocks are assumed to be time-invariant across the full sample.

5 Conclusions

Heteroskedasticity is used as a tool for identifying structural shocks in a number of studies using structural VARs. A basic precondition for identifying the shocks in this context is the time-invariance of the impact effects of the shocks also at times of volatility changes. In this study we present possible tests for time-varying impact effects of the shocks if long-run restrictions are available to identify the shocks and, hence, any identifying restrictions from heteroskedasticity become over-identifying.

Using our tests we consider the question whether stock prices in the U.S. are mainly driven by speculation or by fundamentals. We find that the impact effects of a fundamental shock are larger in the latter part of our sample implying that fundamental shocks have been more important for the stock market since the middle of the 1980s than they were before. This also demonstrates the usefulness of our tests for applied work and the importance of carefully investigating the possibility of time-varying impulse responses at times of volatility changes. Our results also question the practice to assume time-invariant impact effects of the shocks to identify the structural shocks by heteroskedasticity. In any case, it is worth checking that assumption carefully.

A Appendix

A.1 Data

Table 7: Data Description, Sources, and Sample Periods

Variable	Description	Source	Transf.	Sample Period
gdp_t	US Gross Domestic Product (GDPC1)	FRED	log	1947Q1 - 2019Q4
r_t	3-Month Treasury Bill Secondary Market Rate (TB3MS) deflated by US Consumer Price Index (CPIAUCSL)	FRED	log	1947Q1 - 2019Q4
s_t	S&P 500 Index deflated by US Consumer Price Index (CPIAUCSL). The monthly S&P 500 series is obtained from Robert Shiller's website (http://www.econ.yale.edu/ shiller/data.htm). Quarterly data are obtained taking the last month of each quarter.	Robert Shiller's website, FRED	log	1947Q1 - 2019Q4
e_t	Real earnings	Lütkepohl and Velinov (2016) replication codes	log	1947Q1 - 2012Q1

A.2 Details on Computations

A.2.1 Computation of the Wald Statistic $W(\eta_k)$

In this appendix we describe how the Wald statistic in equation (11) is constructed. In particular, we use the following bootstrap to obtain the estimates $\widehat{\Sigma}_{\beta_k}(m)$ and $\widehat{\Sigma}_{\beta_k}(n)$:

1. The estimated residuals from the VECM model (3) are resampled using a regime-specific moving block bootstrap as in Bruns and Lütkepohl (2024) to obtain bootstrap residuals $u_1^{(j)}, \ldots, u_T^{(j)}$, where the superscript (j) denotes the number of the bootstrap sample.

2. Bootstrap data $y_1^{(j)}, \ldots, y_T^{(j)}$ are recursively generated using the VAR representation in (1).

3. A VECM model is fitted to $y_t^{(j)}$ to obtain $\hat{u}_1^{(j)}, \ldots, \hat{u}_T^{(j)}$.

4. The normalised impact effects $\hat{\eta}_k(m)^{(j)}$ and $\hat{\eta}_k(n)^{(j)}$ are obtained minimising (7) subject to the long-run restrictions.

5. Their covariance matrix across bootstrap samples is estimated as $\widehat{\Sigma}_{\hat{\eta}_k}(m) = \text{cov}(\hat{\eta}_k(m)^{(j)})$ and $\widehat{\Sigma}_{\hat{\eta}_k}(n) = \text{cov}(\hat{\eta}_k(n)^{(j)})$.

6. The test statistic is computed as

$$W(\eta_k) = (\hat{\eta}_k(m) - \hat{\eta}_k(n))' \left(\widehat{\Sigma}_{\hat{\eta}_k}(m) + \widehat{\Sigma}_{\hat{\eta}_k}(n) \right)^{-1} (\hat{\eta}_k(m) - \hat{\eta}_k(n)).$$

A.2.2 Estimation of Lag Order and Cointegration Rank

We follow Cavaliere et al. (2018) in jointly determining the lag order p and the cointegration rank r using a Bayesian Information Criterion (BIC) and allowing for heteroskedastic residuals. We include a linear trend in the model. The BIC in its generic form is

$$\text{BIC}(p, r) = -2\text{llik}_T(p, r) + \text{pen}_T(p, r), \tag{16}$$

where $\text{llik}_T(p, r)$ is the log-likelihood of the model depending on (p, r) and $\text{pen}_T(p, r)$ is a penalty term. Their computation proceeds as follows:

1. Define
   - $S_{00}(p) = T^{-1} \sum_{t=1}^{T} \Delta y_t \Delta y_t'$
   - $S_{10}(p) = T^{-1} \sum_{t=1}^{T} \Delta y_t (y_{t-1}', d_t)'$, where $d_t = (1, \ldots, T)$ is a time trend
   - $S_{11}(p) = T^{-1} \sum_{t=1}^{T} (y_{t-1}', d_t)(y_{t-1}', d_t)'$.

2. Compute $\hat{\lambda}_1(p) > \cdots > \hat{\lambda}_K(p)$ the K largest solutions to the eigenvalue problem $|\lambda S_{11}(p) - S_{10}(p) S_{00}(p) S_{01}(p)| = 0$

3. Compute $\pi(p, r) = r(2K - r + 1) + K(K + 2)/2 + K^2(p - 1)$

4. Then we obtain the penalty term $\text{pen}_T(p,r) = \log(T)\pi(p,r)$

5. and the log likelihood $\text{llik}_T(p,r) = -\frac{T}{2}\log(|S_{00}(p)|) - \frac{T}{2}\sum_{i=1}^{r}\log(1-\hat{\lambda}_i(p))$.

The procedure is repeated for the desired number of pairs (p,r) and we choose $(p^{BIC}, r^{BIC}) = \underset{(p,r)}{\text{argmin}}\ \text{BIC}(p,r)$.

A.2.3 Estimation of Structural VARs with Recursive Long-run Restrictions and Cointegration Rank Zero

Using the notation from Section 1 and assuming that the cointegration rank of the VAR model is zero, the long-run effects of the shocks for $m = 1, \dots, M$, are

$$\Upsilon(m) = \Xi B^*(m)$$

with

$$\Xi = \left(I_K - \sum_{i=1}^{p-1}\Gamma_i\right)^{-1}$$

(see equations (4) and (6)). If $\Upsilon(m)$ is lower-triangular, it can be obtained by a Choleski decomposition of

$$\Upsilon(m)\Upsilon(m)' = \Xi B^*(m)B^*(m)'\Xi' = \Xi\Sigma_u\Xi'$$

Since there are only reduced-form quantities on the right-hand side, $\Upsilon(m)$ can be obtained easily and, thus, as Ξ is invertible, we can get $B^*(m)$ as

$$B^*(m) = \Xi^{-1}\Upsilon(m) = \Xi^{-1}\text{chol}(\Xi\Sigma_u(m)\Xi').$$

A simple bivariate model of this type was already considered by Blanchard and Quah (1989) in their seminal study on using restrictions on the long-run effects of the structural shocks for identification.

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