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The Equivalence of Contests*

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Abstract

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1. Introduction

Economic and social interactions in which players expend costly resources in order to win a prize are often portrayed as contests. Since the seminal papers of Tullock (1980) and Lazear and Rosen (1981), a number of contests have been introduced, e.g. Skaperdas (1992), Kaplan et al. (2002), and Baye et al. (2005), to name a few. Each of these studies investigates different aspects of contests such as interdependency between prizes and resource expenditures, endogenous prize valuation, and the effect of spillovers. For example, Skaperdas (1992) describes a contest where the final payoff depends on the residual resources and the prize. Kaplan et al. (2002) study a contest with effort-dependent prizes. Baye et al. (2005) study litigation contests where, depending on the litigation system, players might compensate rivals for a portion of their legal expenditures. This creates either negative or positive spillover effects of one player's expenditure on another.

In this paper we use a Tullock-type contest model to show that intuitively and structurally different contests can be strategically and revenue equivalent to each other. We consider a simple two-player contest, where outcome-contingent payoffs are linear functions of prizes, own effort, and the effort of the rival. Under this structure, we identify strategically equivalent contests that generate the same family of best response functions and, as a result, the same equilibrium effort expenditures and the same revenue. We also show that the two strategically and revenue equivalent contests may yield different equilibrium payoffs.

A number of studies have previously tried to establish common links between different contests in the literature. For example, Che and Gale (2000) provide a link between a rank-order tournament of Lazear and Rosen (1981) and an all-pay auction of Hillman and Riley (1989). Chowdhury (2009) shows the connection between all-pay auctions (Siegel, 2009) and capacity-constrained price contests (Osborne and Pitchik, 1986). Sheremeta et al. (2009) provide a link between a rent-seeking contest of Tullock (1980) and a rank-order tournament. Similarly, Hirshleifer and Riley (1992) show how, with some assumptions on the distribution of noise, an R&D race between two players which is modeled as a rank-order tournament is equivalent to a rent-seeking contest.[1] Finally, Baye and Hoppe (2003) identify conditions under which research tournament models (Fullerton and McAfee, 1999) and patent race models (Dasgupta and Stiglitz, 1980) are strategically equivalent to the rent-seeking contest. These duality results permit one to apply results derived in the rent-seeking contest literature to the innovation, patent race, and rank-order tournament models, and vice versa.

It is important to emphasize that all studies mentioned above establish links between different families of contests, such as all-pay auctions, rent-seeking contests, and rank-order tournaments. The main finding of this paper is conceptually different from findings of the previous studies. In particular, we show that even within the same family of Tullock-type contests, different types of contests might produce the same best response functions and the same revenue. This is an important finding for a number of reasons. First, from the point of view of a contest designer, if a simple contest generates the same revenue as a more complicated contest, then the administrator can reduce operational cost by replacing the more complicated contest with a simpler equivalent contest. Second, a contest designer seeking Pareto improvement may choose a contest that generates the same revenue, incurs the same cost, but results in higher expected payoffs for contestants. Also, it is possible that, although equivalent, two contests can be affected differently by factors such as risk aversion, loss aversion, or joy of winning.

[1] This result was implicitly demonstrated earlier by Loury (1979). Recently, Jia (2007) extended the result of Hirshleifer and Riley (1992), by proving a more general equivalence between the rank-order tournament and the rent-seeking contest. Fu and Lu (2008) also showed that the rent-seeking contest could further include auctions with pre-investment (Tan, 1992).

Therefore, one contest may be preferred over another based on the importance of these external factors. Finally, certain contests may not be feasible to implement in the field due to regulatory restrictions, or due to the possibility of collusion among contestants. However, such restrictions may not apply to other equivalent contests.

2. Theoretical Model

Consider a two-player contest with two prizes. The players are denoted by i and j. Both players value the winning prize as $W > 0$ and the losing prize as $L \geq 0$. To promote incentives, we assume that the winning prize provides higher valuation than the losing prize, i.e. $W > L$. Players simultaneously expend irreversible and costly efforts $x_i \geq 0$ and $x_j \geq 0$. The probability of player i winning the contest is described by a lottery contest success function:

$$p_i(x_i, x_j) = \begin{cases} x_i/(x_i + x_j) & \text{if } x_i + x_j \neq 0 \\ 1/2 & \text{if } x_i = x_j = 0 \end{cases} \tag{1}$$

Contingent upon winning or losing, the payoff for player i is a linear function of prizes, own effort, and the effort of the rival:

$$\pi_i(x_i, x_j) = \begin{cases} W + \alpha_1 x_i + \beta_1 x_j & \text{with probability} \quad p_i(x_i, x_j) \\ L + \alpha_2 x_i + \beta_2 x_j & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{2}$$

where α_1, α_2 are cost parameters, and β_1, β_2 are spillover parameters. [2] We define the contest described by (1) and (2) as $\Gamma(i, j, \Omega)$, where $\Omega = \{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\}$ is the parameter space. All parameters in Ω along with the contest success function (1) are common knowledge for both players. The players are assumed to be risk neutral, therefore, for a given effort pair (x_i, x_j), the expected payoff for player i in contest $\Gamma(i, j, \Omega)$ is:

[2] The generalized contest defined by (1) and (2) covers the majority of contests in the literature. Most Tullock-type contest studies assume the case of two risk-neutral players with lottery contest success function. The linear structure of the model provides the essential intuition for most field applications.

$$E\big(\pi_i(x_i,x_j)\big) = \frac{x_i}{x_i+x_j}\big(W+\alpha_1 x_i+\beta_1 x_j\big) + \frac{x_j}{x_i+x_j}\big(L+\alpha_2 x_i+\beta_2 x_j\big) \qquad (3)$$

Player i's best response is derived by maximizing $E\big(\pi_i(x_i,x_j)\big)$ with respect to x_i:

$$x_i^{BRF} = -x_j + \sqrt{\frac{\{(\alpha_1-\alpha_2)-(\beta_1-\beta_2)\}x_j^2-\{W-L\}x_j}{\alpha_1}} \qquad (4)$$

We restrict the parameters appropriately as specified in Chowdhury and Sheremeta (2009) to obtain a unique symmetric equilibrium:[3]

$$x_i^* = x_j^* = x = \frac{(W-L)}{-(3\alpha_1+\alpha_2)-(\beta_1-\beta_2)} \qquad (5)$$

Note that equilibrium effort (5) depends on α_1, α_2, the difference between β_1 and β_2, and the spread between the winning and the losing prize valuations.[4] All comparative statics results have sound economic interpretations. For example, the equilibrium effort expenditures increase in the value of the winning prize, W, and decrease in the value of the losing prize, L. Similarly, the effort expenditures decrease if the marginal cost of winning, α_1, increases.

3. Equivalent Contests

In this section, we discuss different contests $\Gamma(i,j,\Omega)$ that are strategically and revenue equivalent to each other. Some of these contests are also payoff equivalent. Therefore, we start by providing basic definitions of strategic, revenue, and payoff equivalence.

Definition 1: Contests are strategically equivalent if they generate the same family of best response functions.

Definition 2: Contests are revenue equivalent if they result in the same equilibrium rent dissipation.

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[3] The needed restrictions on the parameters are: $\alpha_1 < 0$, $\alpha_2 \leq 0$, $\beta_2 - \alpha_1 \geq 0$, $-(3\alpha_1+\alpha_2)-(\beta_1-\beta_2) > 0$ and $(5\alpha_1-\alpha_2)-(\beta_1-\beta_2) < 0$.

[4] From (5) one can derive the expected equilibrium payoff as $E^*(\pi) = \frac{(\beta_2-\alpha_1)(W-L)}{-(3\alpha_1+\alpha_2)-(\beta_1-\beta_2)} + L$.

Definition 3: Contests are payoff equivalent if they generate the same expected payoffs.

Note that, according to the definition, strategic equivalence also implies revenue equivalence. The following Proposition formalizes this result.

Proposition: Any strategically equivalent contests $\Gamma(i, j, \Omega)$ are also revenue equivalent.

The proof of the Proposition is trivial. Since (5) is derived from (4), two contests that have the same best response functions (4) bound to exhibit the same equilibrium rent dissipation (5). Obviously, contests that generate the same equilibrium efforts also generate the same revenue, since the revenue of a contest designer is simply a summation of all individual efforts (Baron and Myerson, 1982; Moldovanu and Sela, 2001).

Strategic equivalence is a stronger condition than revenue equivalence because it requires different contests to generate exactly the same best response functions, and thus the same equilibrium efforts. Hence, the concept of strategic equivalence portrays a sufficient condition to achieve revenue equivalence. It is well known that establishing revenue equivalent contests may not be easy, since it requires obtaining the equilibrium first. Strategic equivalence, on the other hand, requires obtaining only the best response functions, without solving for the equilibrium. Given the Proposition, one can find sufficient conditions for two contests to be revenue equivalent by simply comparing the best response functions generated by these contests.[5] The definition of strategic equivalence, however, does not imply payoff equivalence. As we show in the next section, depending on the cost and spillover parameters in Ω, one contest can generate higher payoff than another strategically equivalent contest. Nevertheless, the majority of contests that we will discuss are both strategic/revenue and payoff equivalent.

[5] It is important to emphasize that contests that generate the same revenue do not necessarily generate the same individual efforts. This can happen when players are heterogeneous or, even with homogeneous players, when there are multiple equilibria (Chowdhury and Sheremeta, 2009). Therefore, strategic equivalence always implies revenue equivalence but the opposite is not always true.

When trying to find a strategically equivalent contest to a particular baseline contest, we propose a two-step procedure. First, we derive the best response function of the baseline contest. Second, from the best response function of the baseline contest we derive the restrictions needed for a more general family of contests to generate the same best response functions. This simple and useful procedure is used throughout our analysis. We begin with the original contest of Tullock (1980).

3.1. Original Tullock Contest

In the standard contest defined by Tullock (1980) there is no losing prize and regardless of the outcome of the contest both players completely forgo their efforts. In such a case, $W > 0$, $\alpha_1 = \alpha_2 = -1$, and the other parameters in Ω are zero. The payoff for player i in case of winning or losing is:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ -x_i & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{6}$$

Using our notation, the Tullock contest is defined as $\Gamma(i, j, \{W, 0, -1, -1, 0, 0\})$. The resulting best response function in such a contest for player i is:

$$x_i = -x_j + \sqrt{W x_j}. \tag{7}$$

For a generic contest $\Gamma(i, j, \{W, L, \alpha_1, \alpha_2, \beta_1, \beta_2\})$ to be equivalent to a contest $\Gamma(i, j, \{W, 0, -1, -1, 0, 0\})$ we need to impose the following restrictions: $W - L = W$, $\alpha_1 = -1$, and $\beta_2 - \beta_1 - \alpha_2 = 1$. Such restrictions guarantee that the best response function (4) is exactly the same as the best response function (7). Therefore, by definition these contests are equivalent. This equivalence result incorporates a well known fact in the rent-seeking literature that increase or decrease of the winning and the losing prize valuations by the same amount does not affect the equilibrium. Hence, if we define a contest with positive losing prize $L = \Delta$ and winning prize $W = W + \Delta$ as $\Gamma(i, j, \{W + \Delta, \Delta, -1, -1,0,0\})$, then the best response function and equilibrium expenditures will be the same as in the original Tullock contest.

One particularly interesting case arises when we put further restrictions $L = 0$, $\beta_1 = -1$, and $\alpha_2 = \beta_2 = 0$. In such a contest, $\Gamma(i, j, \{W, 0, -1,0, -1,0\})$, the new payoff function is:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i - x_j & \text{with probability} \quad p_i(x_i, x_j) \\ 0 & \text{with probability} \;\; 1 - p_i(x_i, x_j) \end{cases} \tag{8}$$

Note that in (8), the winner fully reimburses the loser. It can be easily shown that the unique equilibrium for contests defined by (6) and (8) is the symmetric equilibrium with $x_i^* = x_j^* = W/4$. Moreover, the expected payoff in both contests is exactly the same, $E^*(\pi) = W/4$. Therefore, contests (6) and (8) are both strategically and payoff equivalent. This equivalence is surprising, since the two contests are intuitively and structurally very different. In (6) the winner and the loser completely forgo their efforts. On the contrary, in (8) only the winner has to pay all effort expenditures by both contestants.

It is also straightforward to show that the 'input spillover' contest of Chowdhury and Sheremeta (2009), where the effort expended by player *j* partially affects player i and vice versa, is equivalent to the original Tullock contest. The spillover contest can be defined as $\Gamma(i, j, \{W, 0, -1, -1, \beta, \beta\})$, where $\beta \in (-1,1)$ is the input spillover parameter. From (4), one can see that for any value of β, the resulting best response function is exactly same as in (7). Hence, the input spillover contest $\Gamma(i, j, \{W, 0, -1, -1, \beta, \beta\})$ is equivalent to the original Tullock contest $\Gamma(i, j, \{W, 0, -1, -1,0,0\})$. This equivalence is again surprising, given the fact that both contests conceptually are very different. This result suggests that if an R&D competition is modeled as a lottery contest, then the existence of symmetric spillovers may not affect the equilibrium. However, the 'input spillover' contest is not payoff equivalent to the original Tullock contest. Specifically, a positive (negative) spillover provides a higher (lower) payoff to the players than the Tullock contest.

3.2. Modified Tullock-Type Contests

Economists often use modified payoffs in Tullock contests in order to address specific research questions. There are instances in the literature where two different Tullock-type contests are equivalent to each other. In this subsection we briefly discuss some of these examples.

Chung (1996) assumes that the value of the winning prize depends on the total effort expenditures in the contest. A simple linear version of the Chung (1996) model would generate the following payoff function:

$$\pi_i(x_i,x_j) = \begin{cases} W + a(x_i + x_j) - x_i & \text{with probability} \quad p_i(x_i,x_j) \\ -x_i & \text{with probability} \quad 1 - p_i(x_i,x_j) \end{cases} \tag{9}$$

Hence, (9) can be described as $\Gamma(i,j,\{W,0,a-1,-1,a,0\})$, where $a \in (0,1)$, and the best response function is:

$$x_i = -x_j + \sqrt{Wx_j/(1-a)} \tag{10}$$

Lee and Kang (1998) study a contest with externalities. In their model the cost of effort decreases with the total effort expenditures. This contest can be captured by:

$$\pi_i(x_i,x_j) = \begin{cases} W - x_i + b(x_i + x_j) & \text{with probability} \quad p_i(x_i,x_j) \\ -x_i + b(x_i + x_j) & \text{with probability} \quad 1 - p_i(x_i,x_j) \end{cases} \tag{11}$$

Hence, (11) can be described as $\Gamma(i,j,\{W,0,b-1,b-1,b,b\})$, where $b \in (0,1)$, and the best response function is:

$$x_i = -x_j + \sqrt{Wx_j/(1-b)} \tag{12}$$

It is clear that when $a = b$ the best response functions (10) and (12) and the equilibrium effort expenditures in the two contests are exactly the same. This result indicates that some contests with endogenous prizes, as in Chung (1996), are equivalent to contests with externalities, as in Lee and Kang (1998). Also note that, although both contests are strategically equivalent, they are not payoff equivalent. In particular, contest defined by (11) results in higher expected payoff than contest defined by (9), providing a clear Pareto ranking between the two contests. Hence, a benevolent contest designer, such as the government trying to maximize the total social welfare, may opt to choose a contest that elicits the same level of expenditures and, at the same time, results in Pareto improvement.

Next, we consider a 'limited liability' contest introduced by Skaperdas and Gan (1995), where the loser's payoff is independent of the efforts expended. The authors motivate this example by stating that contestants may be entrepreneurs who borrow money to spend on research and development and thus are not legally responsible in case of a loss. The loser of such a contest is unable to repay the loan and goes bankrupt. In such a case, $W > 0$, $\alpha_1 = -1$, and the other parameters in Ω are zero. The payoff is:

$$\pi_i(x_i, x_j) = \begin{cases} W - x_i & \text{with probability} \quad p_i(x_i, x_j) \\ 0 & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{13}$$

The best response function for player i is:

$$x_i = -x_j + \sqrt{x_j^2 + W x_j} \tag{14}$$

Under the symmetric equilibrium we get $x_i^* = x_j^* = W/3$. This contest has one distinctive feature – it is not affected by risk preferences. So, if one were to relax the assumption that players are risk averse, the contest defined by (13) would be unaffected (Skaperdas and Gan, 1995).

For a contest to be equivalent to $\Gamma(i,j,\{W,0,-1,0,0,0\})$ we need to impose the following restrictions: $L=0$, $\alpha_1=-1$, and $\beta_2-\beta_1-\alpha_2=0$. When we impose further restrictions $\alpha_2=-1$, $\beta_2=-1$ and $\beta_1=0$ we obtain a contest with the following payoff function:

$$\pi_i(x_i,x_j)=\begin{cases} W-x_i & \text{with probability} \quad p_i(x_i,x_j) \\ -x_i-x_j & \text{with probability} \quad 1-p_i(x_i,x_j) \end{cases} \tag{15}$$

This contest can be interpreted as a 'full liability' contest, since the loser has to pay in full the expenditures of both players. Note that although (13) is strategically equivalent to (15), a 'full liability' contest is (by definition) more risky than a 'limited liability' contest. In (13) players do not have to worry about what happens in the case of loss, since they are not legally responsible. In contrast, the loser in (15) has to pay the expenditures of both players.[6] Therefore, equivalence between (13) and (15) holds only under the assumption of risk neutrality.

Alexeev and Leitzel (1996) study a 'rent-shrinking' contest $\Gamma(i,j,\{W,0,-1,-1,-1,0\})$, where the winning prize value decreases by the total effort expenditures. An equivalent contest would require the restrictions $\alpha_1=-1$ and $\beta_2-\beta_1-\alpha_2=2$. A 'lazy winner' contest $\Gamma(i,j,\{W,0,-1,-2,0,0\})$ of Chowdhury and Sheremeta (2009), in which the marginal cost of winning ($\alpha_1=-1$) is lower than the marginal cost of losing ($\alpha_2=-2$), definitely satisfies these restrictions. Moreover, the two contests are also payoff equivalent. The equivalence between the 'rent-shrinking' and 'lazy winner' contests enables the designer to achieve the same equilibrium rent dissipation using two alternative contests. Nevertheless, the 'lazy winner' contest is easier to implement and it is less susceptible to the collusion problem mentioned in Alexeev and Leitzel (1996).

[6] As a result, the expected payoff in a 'full liability' contest is $E^*(\pi)=0$ and in a 'limited liability' contest it is $E^*(\pi)=W/3$.

In many cases a contest designer, such as the government, can use different policy tools to implement a certain contest. Using the same procedure as before it can be shown that under certain restrictions, contests with endogenous valuations (Amegashie, 1999), contests with differential cost structure (Chowdhury and Sheremeta, 2009), and contests with taxes (Glazer and Konrad, 1999), are equivalent.[7] This latter equivalence conveys another important message. It shows that the designer can either use policy tools, such as taxes, or contests with alternative cost structure to achieve the same objective. Moreover, since the three contests are not payoff equivalent, a contest designer, such as the government trying to maximize the social welfare, can also achieve a Pareto improvement by choosing a specific contest structure.[8]

3.3. Contest with Complementarities

Next we describe a contest that captures the complementarity between prizes and resource expenditures (Skaperdas, 1992; Garfinkel and Skaperdas, 2000). In such a contest there are two players i and j with limited endowments of E_i and E_j. Both players are competing to win the contest. The winner of the contest receives the sum of resource endowments, $E_i + E_j$, minus the sum of efforts expended by both players, $x_i + x_j$. It is also assumed that the conflict destroys a fraction $(1-\phi) \in (0,1)$ of the total payoff. This contest can be interpreted as a war between two countries, where each country possesses a pool of human capital, E_i and E_j. A portion of the human capital is used as soldiers, x_i and x_j, to fight with the other country for a piece of land.

[7] Glazer and Konrad (1999) study a contest $\Gamma(i,j,\{(1-t)W,0,-(1-t),-1,0,0\})$, in which a part of the rent seeker's non-negative profit is taxed. Amegashie (1999) studies a contest $\Gamma(i,j,\{W,0,-(1-m),-1,0,0\})$, in which the winner's prize value is a linear function of own effort spent. Chowdhury and Sheremeta (2009) study a contest $\Gamma(i,j,\{W,0,\alpha_1,\alpha_2,0,0\})$, in which the marginal cost of winning is lower than the marginal cost of losing, i.e. $|\alpha_1| < |\alpha_2|$. Note that when $\alpha_1 - \alpha_2 = t = m$ and $\alpha_1 = t - 1 = m - 1$ then the three contests are equivalent.

[8] The equilibrium payoff (under the restriction of strategic equivalence) in Glazer and Konrad (1999) is $E^*(\pi) = (1-t)^2\mathrm{W}/(4-3t)$, in Amegashie (1999) it is $E^*(\pi) = (1-t)W/(4-3t)$, and in Chowdhury and Sheremeta (2009) it is $E^*(\pi) = (1-t)\mathrm{W}/(2-3t)$.

The winner then uses the residual human capital, $(E_i - x_i)$ and $\left(E_j - x_j\right)$, as farmers to produce output on the land. To generate such a contest, we need to impose the following restrictions: $W = \phi(E_i + E_j)$, $\alpha_1 = \beta_1 = -\phi$, and the other parameters in Ω are zero. Thus, the payoff function is:

$$\pi_i(x_i, x_j) = \begin{cases} \phi(E_i - x_i) + \phi\left(E_j - x_j\right) & \text{with probability} \quad p_i(x_i, x_j) \\ 0 & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{16}$$

The best response function in such a contest, $\Gamma(i, j, \{\phi\left(E_i + E_j\right), 0, -\phi, -\phi, 0,0\})$, is:

$$x_i = -x_j + \sqrt{(E_i + E_j)x_j}. \tag{17}$$

Although E_i and E_j can be different, the equilibrium efforts for players i and j are the same, $x_i^* = x_j^* = (E_i + E_j)/4$. Note that the equilibrium effort expenditures do not depend on the destruction parameter ϕ, but only on the resource endowments, E_i and E_j (Garfinkel and Skaperdas, 2000).

For the contest to be equivalent to $\Gamma(i, j, \{\phi\left(E_i + E_j\right), 0, -\phi, -\phi, 0,0\})$ we need to impose the following restrictions: $W - L = E_i + E_j$, $\alpha_1 = -\phi$, and $\beta_2 - \beta_1 - \alpha_2 = \phi$. One interesting case arises when we impose further restrictions $W = \phi E_i$, $L = -\phi E_j$, $\beta_2 = \phi$, and $\beta_1 = \alpha_2 = 0$. In such a contest, $\Gamma(i, j, \{\phi E_i, -\phi E_j, -\phi, 0,0, \phi\})$, the payoff function is:

$$\pi_i(x_i, x_j) = \begin{cases} \phi(E_i - x_i) & \text{with probability} \quad p_i(x_i, x_j) \\ -\phi(E_j - x_j) & \text{with probability} \quad 1 - p_i(x_i, x_j) \end{cases} \tag{18}$$

This contest can be interpreted as a 'harmful residual' contest, where the winner gains from his residual human capital, $\phi(E_i - x_i)$, and also uses it to destroy the property of the losing party, $-\phi(E_j - x_j)$. A real life example of such a contest is the 'Scorched Earth' policy used in several war including American Civil War, Boer War, World War II and Gulf War. In these examples, the winning forces destroyed the civil properties of the defeated countries, instead of using them. Such strategy would serve two purposes. First, it directly harms the rivals, and second it indirectly benefits the winners since the rivals lose the resources to fight back in the future. It is again very interesting to note that although contests defined by (16) and (18) are very different in nature, they are strategically equivalent.[9] Moreover, the expected payoff in both contests is exactly the same, $E^*(\pi) = \phi(E_i + E_j)/4$.

4. Discussion

In this paper we define strategically equivalent contest as the contests that generate the same family of best response functions. We derive conditions for strategic equivalence of different contests and show that strategically equivalent contests are also revenue equivalent. A simple two-step procedure is described to identify strategically equivalent contests. Using this procedure, we identify contests that are strategically equivalent to the original Tullock contest, show equivalent contests under prize-effort complementarity, and provide new examples of strategically equivalent contests. This equivalence is an important finding because it demonstrates that different contests can be used to achieve the same objective. We also show that the two strategically equivalent contests may yield different equilibrium payoffs. Hence, a contest designer has the option to choose between contests that elicit the same equilibrium rent dissipation, but have different Pareto ranking.

An important question one needs to address is what are the critical conditions needed for the equivalence to hold. The answer lies in the structure of our model. Following the majority of the rent-seeking contests in the literature, we consider a two-player Tullock-type contest with linear cost and spillover structure under risk neutrality. The simple equivalence results may not

[9] One could also construct a more general model, where, instead of equal destruction rates as in (11), the destruction rate is different for the winner,ϕ, than for the loser, μ.

hold if we relax one or more of these assumptions. First, the assumption of risk neutrality might not hold in many applications and, therefore, one might expect the equivalence not to hold in the case of risk aversion. Second, we assume that probability of winning is defined by a lottery contest success function: $p_i(x_i, x_j) = x_i/(x_i + x_j)$. However, it is likely that the equivalence results of some contests may not hold if a more general contest success function is applied: $p_i(x_i, x_j) = x_i^r/(x_i^r + x_j^r)$, with $r \neq 1$,. We also assume that a contest is between two players. One may expect changes in the equivalence results in the case of more than two players (with single or multiple winners). Finally, there are practical applications when costs are convex and spillovers influence the payoff function non-linearly. A different analysis of equivalence would be required in such cases. Nevertheless, the concept of equivalence and the two-step procedure to obtain equivalent contests will still be relevant for such analysis. Using the two-step procedure one could, for example, find equivalence conditions with n-players, single (multiple) winner(s), risk aversion, and non-linear cost/spillover structure.

This study covers a broad area of the literature and provides an important argument. We acknowledge, however, that the equivalence examples portrayed in this study are only representative and not exhaustive. Hence, our study also demonstrates a need for further theoretical and empirical investigation of equivalent contests. Other than finding out equivalence condition under different structure, one could also design an experiment as in Sheremeta (2009a,b) to test whether equivalence results hold and whether risk aversion, loss aversion, or joy of winning play an important role in sustaining such equivalence. For example, one can design an experiment to test the equivalence between 'full liability' and 'limited liability' contests. As we discussed in Section 3.2, the equivalence between these two contests holds only under the assumption of risk-neutrality. However, it has been documented in laboratory experiments that the majority of people are risk averse (Holt and Laury, 2009), and that risk aversion is correlated with bids in Tullock-type contests (Sheremeta, 2009a). Therefore, one may expect for the ‘limited liability’ contest to generate higher revenue than the ‘full liability’ contest. Similar laboratory experiments can be performed to test other equivalences described in this study. We leave these questions for future research.

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