A Note on Multi-winner Contest Mechanisms *

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Abstract

We consider a multi-winner nested elimination contest in which losers are sequentially eliminated to attain the set of winners. This is a variant of a widely used mechanism introduced by Clark and Riis (1996) that allows one to select the winners sequentially. We show that the nested elimination mechanism becomes equivalent to another popular mechanism suggested by Berry (1993) where the winners are chosen simultaneously.

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1 Introduction

In many contests, in which players expend costly resources in order to win a prize, there are multiple winners. Examples include multiple medals in sports, early bird prizes, set of winners in rent-seeking, set of recipients of research grants - to name a few. In the literature these contests are interchangeably called multi-winner contests (Berry, 1993) or multi-prize contests (Sisak, 2009). We define *multi-winner contests* as contests in which there are more than one prize, but one player may win at most one of them. Most of the examples mentioned above is covered by this definition.

In this note we consider a particular widely applied multi-winner contest mechanism from field in which losers are sequentially eliminated to reach the final set of winners. We show that it is a variant of a highly cited mechanism introduced by Clark and Riis (1996); and is equivalent to another famous mechanism introduced by Berry (1993).

Berry (1993) was the first to analyze multi-winner contests using the framework of rent-seeking (Tullock, 1980). He considers a contest among N players, and $k(< N)$ prizes. He assumes that the probability of a player to win a prize is the sum of efforts expended by any combination of a k-player group *that includes the specified player*, divided by any combination of a k-player group. Hence, the probability that player i wins a prize is:

$$P_i^B(\underline{x}) = \frac{\overbrace{\sum_{j=1}^{k-1} x_j + x_i + \sum_{j=1, j\neq 2}^{k} x_j + x_i + \cdots + \sum_{j=N-k+1}^{N-1} x_j + x_i}^{\binom{N-1}{k-1}\text{times}}}{\underbrace{\sum_{j=1}^{k} x_j + \sum_{j=1, j\neq 2}^{k+1} x_j + \cdots + \sum_{j=N-k+1}^{N} x_j}_{\binom{N}{k}\text{times}}}$$

where $\underline{x}$ is the vector of the efforts, and x_i is the effort of player i.

Clark and Riis (1996) show that this winner selection mechanism inadvertently allows one prize to be allocated according to effort outlays, while the others are allocated independent of the effort outlays. This, in turn, results in free riding among players. They further introduce a nested mechanism in winner selection according to which, players expend effort, then one player is selected as winner using a Tullock (1980) contest success function. Then that player and his efforts are taken out of the calculation and another Tullock contest is run among the remaining $(N-1)$ players using their already expended effort, and another winner is taken out. This procedure is repeated for k times to select the k winners. Here, the probability that player i wins a prize becomes:

$$P_i^{CR}(\underline{x}) = p_i^1 + \sum_{j=1}^{k-1}\left[\prod_{s=1}^{j}(1-p_i^s)p_i^{j+1}\right]$$

where p_i^s is the probability of player i to win the prize at period s. The issue of allocation of prizes being independent of the effort outlays does not arise under this mechanism.

Both of the mechanisms are widely cited by researchers investigating issues in multi-winner contests (see Sisak (2009) for a survey). However, as Clark and Riis (1996) mention, when one allows "*the imperfectly discriminating rent-seeking contest to have several winners, there is no unique method for selecting those winners*". There are other mechanisms that are employed in the field in addition to the two mechanisms introduced above.

One of the popular mechanisms employed in the field is very similar to the one introduced in the Clark and Riis (1996) study. However, instead of selecting-in winners in each nested period, the mechanism selects-out losers. This is common in elimination contests in which the losers are gradually selected out. This includes elimination of losers in sit-and-draw contests (in which contestants draw pictures and the examiners decide upon the winners by eliminating the not-so-good drawings), elimination of job candidates to reach the final set (in which job candidates' CVs are used to eliminate the candidates who do not have a chance), promotional tournaments (where contestants are gradually taken out) etc.

Here we consider a mechanism similar to Clark and Riis (1996) to eliminate $(N-k)$ possible losers in $(N-k)$ elimination periods and show that it turns out to be equivalent to the mechanism suggested by Berry (1993). We then discuss the implications and possible extensions regarding those mechanisms.

2 Model

Consider a contest among N players, and $k(<N)$ prizes. The players exert effort only once, but the winner selection (or loser elimination) process is of multi periods through which

$N-k$ among the N players are eliminated. In each period, one player is eliminated, and when k players are left, the identical prizes (with individual valuation V) are granted to the survivors. A lottery (Tullock, 1980) type contest success function is employed to eliminate losers in every period.

To define the probability of winning a prize, first let I_t be the set of survivors at period t. Also denote the effort level of jth player in I_t by x_j^t.[1] Since one and only one player is dropped out in each period, the number of elements in set I_t is

$$|I_t| = N - t + 1,$$

and the aggregate effort at period t is

$$X^t(I_t) = \sum_{j=1}^{N-t+1} x_j^t$$

Then, conditional on player i has survived the previous periods, the probability that he is eliminated in period t is

$$\begin{aligned} q_i^t(I_t) &= \frac{X^t - x_i}{(X^t - x_1^t) + (X^t - x_2^t) + \cdots + \left(X^t - x_{N-t+1}^t\right)} \\ &= \frac{X^t - x_i}{(N-t)X^t} \end{aligned}$$

As one can easily notice, this probability can be described as a Tullock-type contest *failure* function. Its complementary probability, $1 - q_i^t$, is the probability of winning a prize in Berry's contest where the number of players is $N-t+1$ and that of prizes is $N-t$.

Next, we define the sequence of losers s_l as

$$s_l = \left(n_l^1, n_l^2, \cdots, n_l^{N-k}\right)$$

where n_l^t is the player eliminated in period t according to the schedule of s_l. Since s_l has the same information that sequence $\{I_t\}_{t=1}^{N-k}$ has, we can define the probability of n_l^t being eliminated in period t as

$$p^t(s_l) = q_{n_l^t}^t(I_t)$$

provided that s_l and I_t are consistent; i.e., none of $\left\{n_l^1, n_l^2, \cdots, n_l^{t-1}\right\}$ is in I_t, but $n_l^t \in I_t$.

[1] Note that the effort is expended only once at the start of the contest. Due to the elimination of players in each period, denoting the effort for each period (x_j^t) separately, however, allows ease of notation.

Then, the probability that player i wins a prize is defined as follows:

$$P_i(\underline{x}) = \sum_{s_l \in S_i} \left[\prod_{t=1}^{N-k} p^t(s_l) \right]$$

where $\underline{x}$ is the vector of the efforts, and S_i is the set of all sequences (of length $N-k$) that do not have i in its slots.

Since the valuations are symmetric, we naturally focus on the symmetric case. Let us assume that player i exerts x_i and all the others x_{-i}. Then, for any $s_l \in S_i$,

$$\begin{aligned} p^t(s_l) &= \frac{X^t - x_{n_l^t}}{(N-t)X^t} \\ &= \frac{(N-t-1)x_{-i} + x_i}{(N-t)\left[(N-t)x_{-i} + x_i\right]}, \end{aligned}$$

and

$$\begin{aligned} \prod_{t=1}^{N-k} p^t(s_l) &= \frac{(N-2)x_{-i} + x_i}{(N-1)\left[(N-1)x_{-i} + x_i\right]} \frac{(N-3)x_{-i} + x_i}{(N-2)\left[(N-2)x_{-i} + x_i\right]} \cdots \frac{(k-1)\,x_{-i} + x_i}{k\left[kx_{-i} + x_i\right]} \\ &= \frac{(k-1)\,x_{-i} + x_i}{(N-1)(N-2)\cdots k\left[(N-1)x_{-i} + x_i\right]} \end{aligned}$$

Noting that the number of elements in S_i is

$$|S_i| = \frac{(N-1)!}{(k-1)!}$$

which is the number of cases to choose $N-k$ losers among $N-1$ players (player i is already chosen as a winner), we can write the probability of player i winning a prize as

$$\begin{aligned} P_i(\underline{x}) &= \frac{(N-1)!}{(k-1)!} \times \frac{(k-1)\,x_{-i} + x_i}{(N-1)(N-2)\cdots k\left[(N-1)x_{-i} + x_i\right]} \\ &= \frac{(k-1)\,x_{-i} + x_i}{(N-1)x_{-i} + x_i} \end{aligned}$$

This contest success function is identical with the one suggested by Berry (1993).[2] Hence, the equilibrium effort and the corresponding comparative statics are also the same as the ones in Berry (1993).

[2] See the Appendix for further elaboration.

3 Discussion

We consider a multi-winner nested elimination contest in which losers are sequentially eliminated to attain the set of winners. This mechanism incorporates an array of real life contests and is similar to the one by Clark and Riis (1996), who consider sequential acceptance of winners. We show that theoretically this mechanism is equivalent to the simultaneous selection mechanism of Berry (1993). Since the theoretical results predict different rent-dissipation among winner selection mechanisms, the equivalence result allows contest designers to implementation the appropriate mechanism in accordance to their objectives.

It may be possible in the future for one to introduce other popularly employed mechanisms and compare them with the existing ones. It is also possible to introduce risk aversion, player asymmetry, and prize asymmetry within this structure. Finally, the existing results provide clear ranking of rent-dissipation among multi-winner mechanisms, but it would be interesting to investigate whether the theoretical benchmark results still hold behaviorally. Very little experimental research had been carried out in the area of multi-winner contests (see Dechenaux et al. (2012) for a comprehensive survey), and one obvious first attempt can be to test and compare these three mechanisms in the laboratory.

References

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[3] Dechenaux, E., Kovenock, D., and Sheremeta. R. M. (2012). A survey of experimental research on contests, all-pay auctions, and tournaments. Chapman University Working Paper.

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APPENDIX

Clark and Riis (1996) showed that the contest success function of Berry (1993) can be rewritten as

$$\begin{aligned} P_i^B(\underline{x}) &= \frac{x_i}{X} + \left(1 - \frac{x_i}{X}\right) \frac{k-1}{N-1} \\ &= \frac{(k-1)X + (N-k)x_i}{(N-1)X} \end{aligned}$$

where X is the sum of all efforts. When player i exerts x_i and all the others x_{-i}, the probability becomes

$$\begin{aligned} P_i^B(\underline{x}) &= \frac{(k-1)\left[(N-1)\, x_{-i} + x_i\right] + (N-k)x_i}{(N-1)\left[(N-1)\, x_{-i} + x_i\right]} \\ &= \frac{(k-1)\, x_{-i} + x_i}{(N-1)x_{-i} + x_i} \end{aligned}$$

which is identical with the probability of winning a prize in the sequential-elimination contest.