

Time series non-linearity in the real growth / recession-term spread relationship, some evidence from the UK

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Abstract

This paper examines the existence of time series non-linearity in the real output growth / recession-term spread relationship. Vector Autoregression (VAR), Threshold VAR (TVAR), Structural break VAR (SBVAR), Structural break threshold VAR (SBTVAR) are applied in the analysis. The in-sample results indicate there are non-linear components in this relationship. And this non-linearity tend to be caused by structural breaks. The best in-sample model also shows its robustness on arrival of new information in the out-of-sample tests. We find evidence the model with only structural break non-linearity outperform linear models in 1-quarter, 3-quarter and 4-quarter ahead forecasting.

Keywords: Term spread, Non-linearity, SBTVAR, Forecast, Real growth, Recession, UK

1. Introduction

Exploring the answer of question about ‘How the information from the yield curve will influence the future real economic activities?’ are quite popular since early 1990s’. The intuition of these studies are, agents in the market will invest on assets base on the expectation of the economy, so that the price changing contains useful information about future economic growth. In bond market, these behaviour will lead to a shape changing of the term structure of interest rates. Therefore, as the simplest form of term structure of interest rates, term spread becomes a valid agent to investigates the theory.

Numerous studies applying various data sets and models try to understand how well the term spread explains and forecasts the output growth and recessions. Evidence show that term spread is a reliable predictor under linear analysis (Fama, 1990; Mishkin, 1990; Estrella and Hardouvelis, 1991; Zagaglia, 2006; Bordo and Haubrich, 2008). Nonetheless, the prediction power varies in different economies. It is a valid predictor in the UK and Germany as well as US from most of the literature (Jorion and Mishkin, 1991;

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Harvey, 1991; Estrella and Mishkin, 1997; Plosser and Rouwenhorst, 1994; Bernard and Gerlach, 1998). From 2000, researchers start to look at this relationship from a non-linear perspective. Galbraith and Tkacz (2000) find the evidence of non-linearity in this relationship in the US and Canada. Venetis et al. (2003) discovered threshold effects in the US, UK and Canada in the output growth-yield spread relationship. And Duarte et al. (2005) find significant nonlinearity exists in the output growth-yield spread relation in euro area as well as the US. Galvão (2006) uses Structural break threshold VAR showing models with structural break and thresholds outperform linear models in recession forecasting in the US sample.

In the existing literature, UK has not been tested comprehensively. This allows me to make several contributions to the literature on the output growth / recession-term spread relationship. Firstly, I choose UK as the target country to investigate the relationship between yield spread and real economic activities involving the time series non-linearity. Secondly, the presenting research is conducted with data containing most recent Financial Crisis and tested the influence of this big recession to the relationship. Thirdly, I apply VARs with non-linearity to forecast future real GDP growth as well as recessions. Fourthly, the paper applied 2 more non-linear model and various autoregressive orders in the nest, which is more comprehensive compared to the Galvão (2006)'s research. Last but not least, this study successfully identifies the non-linearity of real growth-term spread relationship in the UK.

UK GDP at constant prices, 3-month UK government bond and 10-year UK government benchmark are chosen quarterly as data from the period between the first quarter of 1979 and the first quarter of 2013.

I applied VARs, Threshold VARs with one (TVAR) or two thresholds (2TVAR), Structural break VARs with one (SBVAR) or two breaks (2SBVAR), Structural Break Threshold VARs with one break and one threshold (SBTVARc) as well as Structural Break Threshold VARs with one break and one threshold in each broken regime (SBTVAR) into real growth-term spread equation. The in-sample results confirm the existence of the nonlinearity and it tend to be structural break. The out-of-sample results show the robustness of the structural break model on arrival of new information. And they present a superior performance against the linear models in 1-quarter, 3-quarter and 4-quarter ahead forecasting.

The rest of this paper is structured as follows: Section 2 is a brief literature review of yield curve forecasting. In Section 3, the structural break threshold VARs and introduction of methods applied for recession forecasting are presented. Section 4 gives the data discription. Section 5 discusses the results and Section 6 concludes.

2. Literature Review

Information in the yield spread contains forecasting ability has been studied widely in the literature. Wheelock and Wohar (2009) did a comprehensive survey regarding

the ability of term spread forecasting output growth and recessions. The survey covers 18 papers focus on growth forecasting and 13 focus on recessions forecasting. All of the paper confirm the forecast ability of the term spreads. In terms of output growth forecasting, Harvey (1988; 1989; 1991) brought this idea and examined in G-7 countries, and confirm the forecast ability of term spread as a leading indicator. Later, Estrella and Hardouvelis (1991), Plosser and Rouwenhorst (1994), Estrella and Mishkin (1997), Dotsey (1998) and Estrella et al. (2003) start to use uni-variate or multi-variate linear models to examine the real growth-term spread relationship.

In 2000, Galbraith and Tkacz (2000) uses nonlinear models and the results show the significance of forecast ability of the yield spread as well as the non-linear behaviour in the relationship. And this bring the research into a new chapter. Tkacz (2001) applied Neural networks model on Canada data shows a greater forecast ability in 4-quarter ahead forecasting than in 1-quarter ahead forecasting. Venetis et al. (2003) conduct nonlinear procedures including smooth nonlinear transition models, regime-switching models and time-varying models using US, UK and Canada data shows that threshold effect exists in the yield spread-output growth relationship. Duarte et al. (2005) use change point model and nonlinear threshold model find that nonlinear model outperforms linear model and spreads achieve a better performance to predict output growth when output growth has slowed. Giacomini and Rossi (2006) present the evidence of structural breaks in US yield spread-output growth relationship. In Benati and Goodhart (2008)'s research, they find the forecasting ability of yield spread varies in different period of time by conducting time-varying parameters VARs. Regarding the recessions forecasting, Probit models are widely used. Estrella and Hardouvelis (1991), Dotsey (1998), Estrella and Mishkin (1998) prove the usefulness of yield spread in recession forecasting using similar US data set. Bernard and Gerlach (1998) and Ahrens (2002) test and present the significance of yield spread as a leading indicator in eight industrialized countries. And Structural break threshold VARs are delivered to predict recession by Galvão (2006) and it suggests 2-quarter ahead forecasts has the best performance in the US. According to the literature above, it is fair to say there are non-linearity in the yield spread-output growth and yield spread-recession relationships in US. However, the literature that examines UK are very limited, and the research have been done mostly use data before 2007. It is important to know whether the most recent 2008 to 2010 recession has altered these relationships.

In this paper, a more comprehensive Structural Break Threshold VARs are applied to examine the existence and influence of non-linearity in the real growth / recession relationship. UK quarterly data until 2013q1 is used to test the consistency of the models when extreme event happens.

3. Methodology

3.1. Structural Break Threshold VARs

Structural break threshold VARs are combinations of Threshold VARs and Structural break VARs. Threshold VARs are piecewise linear models with different autoregressive matrices in each regime, determined by a transition variable (one of the endogenous variables), a delay and a threshold (Tsay, 1998). Structural break models also divide the sample into two or more regimes but they are determined by one or more break-points and are not recurrent, allowing different dynamics before and after the break. Although non-linear models can capture some characteristics of structural break models ((Koop and Potter, 2000, 2001; Carrasco, 2002), it may be the case that the break also implies changes in the parameters that determine the non-linearity. Univariate time-varying smooth transition models have been proposed by Lundbergh et al. (2003) and have been applied to capture changes in seasonality of industrial production by van Dijk et al. (2003). Unlike time-varying parameters models, structural break threshold VARs are able to identify the break point from one regime to another so that one can analyse the cause of the changes.

Define x_t as a $m \times 1$ vector of m endogenous variables $\mathbf{x}_t = (x_{1t}, x_{2t}, \dots, x_{mt})'$ and define the $m \times (mp + 1)$ matrix, $\mathbf{x}_{t-1} = (1, x_{t-1}, \dots, x_{t-p}$ where p is the autoregressive order. A threshold VAR with one threshold (r) with a delay (d) can be written as:

$$\mathbf{x}_t = (\mathbf{x}_{t-1}\beta_1)I_{t-d}(r) + (\mathbf{x}_{t-1}\beta_2)(1 - I_{t-d}(r)) + u_t \quad (1)$$

where r should be allocated in one one m variables before the estimation. In the same manner, a structural break VAR with one break point (τ) can be written as:

$$\mathbf{x}_t = (\mathbf{x}_{t-1}\beta_1)J_t(\tau) + (\mathbf{x}_{t-1}\beta_2)(1 - J_t(\tau)) + u_t \quad (2)$$

A structural break threshold VAR with one break point and one threshold in each structural break regime can be written as:

$$\begin{aligned} \mathbf{x}_t = & [(\mathbf{x}_{t-1}\beta_1)I_{z,t-d_1}(r_1) + (\mathbf{x}_{t-1}\beta_2)(1 - I_{z,t-d_1}(r_1))]J_t(\tau) + \\ & [(\mathbf{x}_{t-1}\beta_3)I_{z,t-d_2}(r_2) + (\mathbf{x}_{t-1}\beta_4)(1 - I_{z,t-d_2}(r_2))](1 - J_t(\tau)) + u_t \end{aligned} \quad (3)$$

where $I_{z,t-d_i}(r_i)$ is an indicator function which depends on a transition variable z . For a threshold r_i and a delay d_i ,

$$I_{z,t-d_i}(r_i) = \begin{cases} 1 & \text{if } (z_{t-d_i} \leq r_i) \\ 0 & \text{if } (z_{t-d_i} > r_i) \end{cases}$$

and $J_t(\tau)$ is another indicator function which depends on a break-point τ ,

$$J_t(\tau) = \begin{cases} 1 & \text{if } (t \leq \tau) \\ 0 & \text{if } (t > \tau) \end{cases}$$

β_i is a $(mp + 1) \times m$ matrix of parameters. u_t is a $m \times 1$ vector of error term. To estimate the threshold VAR models, structural break VAR models and structural break threshold VAR models, there are three methods. They are conditional least square which is suggested by Tsay (1998), maximum likelihood which is provided by Hansen and Seo (2002), and maximum likelihood estimator that allows difference variance in each regime given by Galvão (2006). The conditional least square estimation applies a grid search in part of the sample of threshold and delay and the estimator should be the one minimizes the sum of squared residuals. The sum of squared residuals can be calculated by number of observations times the estimated covariance matrix of residuals for any given threshold. There is a limit on the sample in each regime for searching, and a proportion of π at either end of the data is excluded. And $0 < \pi < 1$. From the literature, 0.10 (Clements and Galvão, 2004) and 0.15 (Andrews, 1993) are usually chosen. Therefore, the conditional least square estimators $(\hat{r}_1, \hat{r}_2, \hat{\tau})$ can be obtained by:

$$\min(T * \text{trace}(\hat{\Sigma}(r_1, r_2, \tau))) \quad \forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u$$

And for maximum likelihood estimator, it is calculated based on assuming error term being normal distributed. Similar to the approach of conditional least square, the estimator is obtained by a grid search in part of the sample in order to minimize $\log(\det(\hat{\Sigma}(r)))$. Therefore, the maximum likelihood estimators $(\hat{r}_1, \hat{r}_2, \hat{\tau})$ can be obtained by:

$$\min(\log(\det(\hat{\Sigma}(r_1, r_2, \tau)))) \quad \forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u$$

Both of the estimators are based on the assumption that the covariance matrices are same for each regime. However, in practice, especially when applying macroeconomic data, the variances are different from each regimes. In order to allow regime-switching variances, Galvão (2006) suggests in a typical SBTVAR (contains one break-point and one threshold in each break period) which has four separated regimes, the maximum likelihood estimator with regime-switching variances $(\hat{r}_1, \hat{r}_2, \hat{\tau})$ can be obtained by:

$$\min \left(\begin{array}{l} \frac{T_1}{2} \log(\det(\hat{\Sigma}_1(r_1, r_2, \tau))) + \frac{T_2}{2} \log(\det(\hat{\Sigma}_2(r_1, r_2, \tau))) \\ \frac{T_3}{2} \log(\det(\hat{\Sigma}_3(r_1, r_2, \tau))) + \frac{T_4}{2} \log(\det(\hat{\Sigma}_4(r_1, r_2, \tau))) \end{array} \right)$$

$$\forall \quad r_l \leq r_1 \leq r_u, r_l \leq r_2 \leq r_u, \tau_l \leq \tau \leq \tau_u$$

In this paper the maximum likelihood estimator with regime-switching variances is applied in estimating the sample.

3.2. Forecasting Recessions

The definition of recession in this paper is adopted the National Bureau of Economic Research (NBER) measurement and it is at least two consecutive negative real economic growth will be considered as experiencing recession.

The probability of the predicting recession is calculated by using estimated VARs simulating future growth. And it is the proportion of number of events which have two consecutive negative real growth over total simulated events. This procedure is first suggested by Anderson and Vahid (2001).

Define $X^{t-1} = \{x_{t-1}, x_{t-2}, \dots, x_1\}$ as the history of x_t and $x_t = f(X^{t-1}; \Gamma) + u_t$ as the forecasting model where Γ is the matrix of parameters, in this paper, they are thresholds and breaks. u_t is i.i.d. with $\text{Var}(u_t) = \Sigma$. For the given value of $\hat{\beta}$ and $\hat{\Sigma}$, I am conducting a forecast of pseudo sequence value for $\{x_t, x_{t+1}, x_{t+2}, x_{t+3}, x_{t+4}\}$. The sequence is obtained by bootstrapping \hat{u}_t for given X^{t-1} and $\hat{\beta}$. Therefore, \hat{X}^t can be formed. Followed by a new draw of u_{t+1} from the residuals and employed to calculate x_{t+1} , given \hat{X}^t and $\hat{\beta}$ so that X^{t+1} is formed. The procedure continues until the sequence $S_1 = \{\hat{x}_t, \hat{x}_{t+1}, \hat{x}_{t+2}, \hat{x}_{t+3}, \hat{x}_{t+4}\}$ has been generated. And then repeat this procedure for 2000 times. The probability of the recession h -quarter ahead is the proportion of these 2000 sequences in which the $x_t + \hat{h} - 1, x_{t+\hat{h}}$ consecutive negative occurs.

In terms of threshold VARs and structural break VARs, the model can be transformed as $x_t^j = f^j(X^{t-1}; \Gamma^j) + u_t^j$, where $j = 1, 2$ for the two regimes. While in the case of structural break threshold VARs, $j = 1, 2, 3, 4$ for the four regimes.

4. Data

The real GDP data used in this paper is ‘‘Gross domestic product(GDP) expenditure approach, at constant price, Seasonal adjusted’’ (y_t). The GDP growth is calculated as follows:

$$\Delta y_t = 100 * \ln(y_{t+4}) - \ln(y_t)$$

For the term spread I applied ‘‘UK Yield 10-Year Central Government Securities’’ (lr_t) minus ‘‘UK Yield Three-Month Treasury Bill’’ (sr_t) The spread is calculated as follows:

$$S_t = lr_t - sr_t$$

All the data are quarterly data from 1979q1 to 2013q1 and they are collected from Datastream.

5. Results

5.1. In-sample Estimation

In order to find the best estimation, I conduct VAR(1), VAR(2) and VAR(3) process with times series non-linearity. 7 models including VARs, Threshold VARs with one (TVAR) or two thresholds (2TVAR), Structural break VARs with one (SBVAR) or two breaks (2SBVAR), Structural Break threshold VARs with one break and one threshold (SBTVARc) and threshold VARs with one break and one threshold in each broken regime (SBTVAR) are estimated. For those threshold models the thresholds and delays will be chosen from the yield spreads.

5.1.1. In-sample Real Growth Estimation

The estimated parameters are shown in Table 1 to Table 6. In which, Table 1 to Table 3 show the delays, thresholds and break parameters as well as the information criteria. Table 4 to Table 6 show the estimated coefficients in these models. Models will be picked by comparing the information criteria and AIC is applied in this research.

Table 1: VAR(1) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		4	4			4	4 4
\hat{r}		2.13	0.88 2.13			2.14	2.20 1.70
$\hat{\tau}$				1988q4	1986q1 1991q3	1991q1	1991q1
						3.10	3.10
σ_Y^2	1.12	1.68	0.69	2.27	2.86	1.23	1.23
		1.22	0.91	1.53	1.84	0.55	0.55
			2.20		0.68	0.49	0.49
						0.92	0.92
σ_S^2	0.64	0.64	1.58	0.76	0.34	0.24	0.24
		0.24	0.64	0.22	0.78	0.58	0.58
			0.24		0.20	0.14	0.14
T	133	50 83	27 22 84	31 102	20 22 91	17 23 33 60	18 22 28 65
AIC	-32.79	-34.05	-39.89	-46.25	-52.38	-80.39	-81.89

Note: Sample period is from 1980Q1 to 2012Q1. σ_Y^2 and σ_S^2 are the estimated variance of output-spread equations for each regime with T observations

According to the VAR(1) results (Table 1), it shows a increasing goodness-of-fit by introduce more regimes. Generally speaking models only with structural break(s) are better than the ones with threshold(s). Cross comparing all the AIC results from Table 1 to Table 3, 2SBVAR(2) gets the best result with a score of -122.16. This means there are 2 structural breaks in the real growth-term spread relationship. They are the first quarter of 1986 and the third quarter of 1991. In association with the results in Table 5 column 11 and 12, the real growth dependent its first lag dropped after 1986q1 while increase significantly after 1991q3. The dependence of second lag of spread is increasing through both the breaks. while for the first lag of spread is increasing through the first break and dropping after the second break. The first break could be related to the expectation changing of people after the whole UK economy has been fully recovered. while the second break could be associated with the government's inflation targeting policy which altered people's expectation using the information in the term spread and let people foresee a longer period.

From Table 1 to 3, it is important to note that a increasing the lag order of the model will increase models goodness-of-fit at first and then decrease. For VAR models' AICs are decreasing by increasing the lag orders. (which will peak at VAR(5)). So do the TVAR and the 2TVAR. While for SBVAR, 2SBVAR, SBTVARc, SBTVAR, AICs start

Table 2: VAR(2) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		4	4			4	1 2
\hat{r}		2.47	0.92 2.13			2.14	1.04 0.87
$\hat{\tau}$				1989q2	1986q1 1991q3	1991q1	1991q1
						3.71	3.71
σ_Y^2	1.02	1.54	0.57	2.06	2.52	1.26	1.26
		1.03	0.79	1.13	1.21	0.90	0.90
			0.94		0.66	0.69	0.69
						0.35	0.35
σ_S^2	0.61	0.42	1.29	0.40	0.31	0.12	0.12
		0.24	0.56	0.17	0.37	0.30	0.30
			0.23		0.16	0.16	0.16
T	133	57 76	29 20 84	33 100	20 22 91	17 23 33 60	9 31 20 73
AIC	-42.37	-43.61	-90.56	-105.99	-122.16	-110.50	-103.74

Note: Sample period is from 1980Q1 to 2012Q1. σ_Y^2 and σ_S^2 are the estimated variance of output-spread equations for each regime with T observations

Table 3: VAR(3) Estimated Parameters

	VAR	TVAR	2TVAR	SBVAR	2SBVAR	SBTVARc	SBTVAR
d		2	2			4	1 4
\hat{r}		1.99	1.99 4.01			1.05	3.39 0.87
$\hat{\tau}$				1989q2	1986q1 1991q2	1991q1	1991q1
						1.61	1.61
σ_Y^2	0.92	1.15	1.15	1.98	2.43	0.98	0.98
		0.69	0.69	1.03	1.15	0.24	0.24
			0.49		0.57	0.30	0.30
						0.34	0.34
σ_S^2	0.55	0.52	0.19	0.39	0.19	0.16	0.16
		0.30	0.37	0.17	0.36	0.29	0.29
			0.17		0.16	0.14	0.14
T	133	48 85	48 63 22	33 100	20 21 92	8 32 23 70	32 8 20 73
AIC	-65.19	-88.92	-94.48	-103.03	-119.62	-107.89	-87.82

Note: Sample period is from 1980Q1 to 2012Q1. σ_Y^2 and σ_S^2 are the estimated variance of output-spread equations for each regime with T observations

to drop after lag orders being increased to 3. This could be led by the parsimonious problem. There are limited observations in some regimes.

Table 4: VAR(1) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVARc		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.203	0.355	-0.206	0.331	0.503	0.421	0.279	0.695	0.251	0.705	-0.403	1.135	-0.355	1.116
	$\beta_{\Delta y, t-1}$	0.896	-0.017	1.019	0.059	0.828	0.003	0.828	0.213	0.788	0.157	0.915	0.454	0.931	0.447
	$\beta_{S, t-1}$	0.008	0.871	-0.120	0.806	-0.110	0.797	0.101	0.479	-0.003	0.333	0.109	-0.046	0.115	-0.049
Regime 2	constant			0.610	0.041	-2.405	0.038	0.123	0.266	0.687	1.757	0.931	0.215	0.873	0.237
	$\beta_{\Delta y, t-1}$			0.867	-0.040	1.456	0.187	0.913	-0.047	0.600	-0.340	0.720	-0.037	0.720	-0.036
	$\beta_{S, t-1}$			-0.049	0.972	0.336	0.843	0.009	0.933	0.310	0.914	0.094	0.921	0.102	0.918
Regime 3	constant					0.603	0.061			0.130	0.223	-0.241	0.427	-0.446	0.301
	$\beta_{\Delta y, t-1}$					0.868	-0.041			0.921	-0.040	1.056	-0.078	1.114	-0.029
	$\beta_{S, t-1}$					-0.048	0.969			0.004	0.936	-0.182	1.022	-0.186	1.097
Regime 4	constant											0.248	-0.165	0.280	-0.154
	$\beta_{\Delta y, t-1}$											0.903	-0.033	0.897	-0.037
	$\beta_{S, t-1}$											0.017	1.024	0.004	1.024

Table 5: VAR(2) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVAR _c		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.349	0.395	0.216	0.350	0.774	0.695	0.473	0.949	0.602	1.092	0.184	1.502	0.646	1.364
	$\beta_{\Delta y, t-1}$	1.151	-0.055	1.024	-0.087	1.301	0.031	0.777	0.015	0.757	-0.094	0.692	0.467	0.421	0.277
	$\beta_{\Delta y, t-2}$	-0.287	0.043	-0.102	0.148	-0.566	-0.122	0.127	0.318	0.073	0.365	0.132	-0.060	-0.264	0.114
	$\beta_{S, t-1}$	0.127	1.025	0.155	0.930	-0.255	0.933	0.215	0.549	0.128	0.354	0.374	0.046	1.649	0.521
	$\beta_{S, t-2}$	-0.147	-0.174	-0.378	-0.191	-0.130	-0.234	-0.267	-0.306	-0.404	-0.379	-0.466	-0.251	-0.535	-0.364
Regime 2	constant			0.611	0.198	-0.238	-0.160	0.500	0.301	0.568	1.936	0.844	0.278	-0.289	-0.689
	$\beta_{\Delta y, t-1}$			1.276	-0.068	1.218	-0.423	1.509	-0.053	0.486	-0.189	0.665	0.012	0.771	-0.016
	$\beta_{\Delta y, t-2}$			-0.435	0.022	-0.067	0.657	-0.677	0.014	0.086	-0.125	-0.050	0.022	0.234	0.384
	$\beta_{S, t-1}$			-0.143	1.192	2.175	1.132	-0.297	1.399	0.269	0.942	-0.026	1.017	0.331	1.032
	$\beta_{S, t-2}$			0.096	-0.249	-2.651	-0.330	0.237	-0.485	0.122	-0.126	0.295	-0.215	-0.258	-0.372
Regime 3	constant					0.725	0.138			0.473	0.256	1.190	0.586	1.890	0.495
	$\beta_{\Delta y, t-1}$					1.115	0.012			1.562	-0.035	1.490	-0.095	0.877	0.032
	$\beta_{\Delta y, t-2}$					-0.261	-0.048			-0.725	0.001	-0.852	-0.027	-0.464	-0.129
	$\beta_{S, t-1}$					0.124	1.107			-0.292	1.373	-0.190	1.557	0.049	1.230
	$\beta_{S, t-2}$					-0.197	-0.162			0.235	-0.452	-0.098	-0.705	0.067	-0.775
Regime 4	constant											0.443	-0.032	0.334	0.261
	$\beta_{\Delta y, t-1}$											1.547	0.011	1.629	-0.030
	$\beta_{\Delta y, t-2}$											-0.690	-0.043	-0.779	0.001
	$\beta_{S, t-1}$											-0.175	1.190	-0.386	1.413
	$\beta_{S, t-2}$											0.135	-0.200	0.365	-0.490

Table 6: VAR(3) Estimated coefficients

		VAR		TVAR		2TVAR		SBVAR		2SBVAR		SBTVARc		SBTVAR	
		Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S	Δy	S
Regime 1	constant	0.367	0.407	0.495	1.105	0.495	1.105	0.297	0.705	0.607	1.074	1.045	2.071	0.294	0.728
	$\beta_{\Delta y, t-1}$	1.102	-0.096	0.952	-0.182	0.952	-0.182	0.812	0.064	0.752	-0.092	0.820	1.669	0.773	0.181
	$\beta_{\Delta y, t-2}$	-0.029	0.260	-0.266	-0.014	-0.266	-0.014	0.189	0.368	0.195	0.466	-0.450	-4.030	-0.296	0.118
	$\beta_{\Delta y, t-3}$	-0.240	-0.203	0.115	-0.029	0.115	-0.029	-0.181	-0.198	-0.164	-0.144	0.301	1.478	0.078	-0.141
	$\beta_{S, t-1}$	0.177	1.068	0.424	0.635	0.424	0.635	0.318	0.678	0.177	0.406	0.009	-0.050	0.589	0.616
	$\beta_{S, t-2}$	-0.377	-0.378	-0.683	0.666	-0.683	0.666	-0.334	-0.417	-0.320	-0.315	0.212	-0.214	-0.083	-0.193
	$\beta_{S, t-3}$	0.205	0.183	0.015	-0.328	0.015	-0.328	0.130	0.196	-0.082	-0.054	-0.485	0.639	-0.016	0.056
Regime 2	constant			0.546	0.402	0.533	-0.118	0.516	0.319	0.974	2.524	-0.218	0.566	3.505	-0.563
	$\beta_{\Delta y, t-1}$			1.343	-0.250	1.438	-0.094	1.459	-0.048	0.370	-0.259	0.875	0.008	0.791	0.061
	$\beta_{\Delta y, t-2}$			-0.246	0.487	-0.468	0.172	-0.569	0.008	0.185	0.121	0.135	0.418	0.544	0.232
	$\beta_{\Delta y, t-3}$			-0.234	-0.296	-0.099	-0.109	-0.067	0.000	-0.264	-0.441	-0.216	-0.335	-0.466	-0.311
	$\beta_{S, t-1}$			-0.217	1.329	-0.240	1.297	-0.238	1.352	0.449	1.039	0.515	0.840	-0.453	1.009
	$\beta_{S, t-2}$			-0.148	-0.730	0.101	-0.262	0.065	-0.351	-0.424	-0.756	-0.690	-0.418	-0.276	-0.793
	$\beta_{S, t-3}$			0.301	0.297	0.066	0.000	0.115	-0.094	0.637	0.715	0.479	0.225	-0.027	0.891
Regime 3	constant					-0.665	3.120			0.438	0.249	2.713	-0.485	3.111	-0.315
	$\beta_{\Delta y, t-1}$					1.269	-0.328			1.566	0.008	1.111	0.153	0.754	0.023
	$\beta_{\Delta y, t-2}$					-0.192	0.684			-0.744	-0.079	-0.572	-0.320	-0.186	-0.183
	$\beta_{\Delta y, t-3}$					-0.230	-0.640			0.019	0.045	-0.343	0.369	-0.486	0.316
	$\beta_{S, t-1}$					-0.349	0.614			-0.232	1.343	-0.449	1.548	-0.604	1.511
	$\beta_{S, t-2}$					-0.037	-0.749			0.053	-0.316	-0.001	-0.466	0.368	-0.402
	$\beta_{S, t-3}$					0.598	0.529			0.132	-0.107	-0.025	-0.224	-0.295	-0.261
Regime 4	constant											0.223	0.108	0.219	0.108
	$\beta_{\Delta y, t-1}$											1.645	-0.042	1.723	-0.002
	$\beta_{\Delta y, t-2}$											-0.871	0.022	-0.998	-0.052
	$\beta_{\Delta y, t-3}$											0.098	-0.019	0.154	0.021
	$\beta_{S, t-1}$											-0.184	1.257	-0.194	1.270
	$\beta_{S, t-2}$											0.117	-0.259	0.085	-0.249
	$\beta_{S, t-3}$											0.073	-0.035	0.112	-0.060

5.1.2. In-sample Recession Estimation

From Figure 1 show the models' in-sample estimation of recessions. The shadows show the of real recession situation. From the graphs we can see all the models can capture the three major recessions in the sample period (1980q1 to 2013q1). Watching closely, the graphs show that the model with threshold model the recession period smoothly while bumpy during the non-recession period. In the contrast, models with only structural break show some volatility during the recession period while smoother in the non-recession period. Linear models (VARs) show less stable in both periods. All the models discover a increasing probability of recession after 2010. While by increasing the lag order, we can see more detail movements, however, models are much smoother in non-recession period.

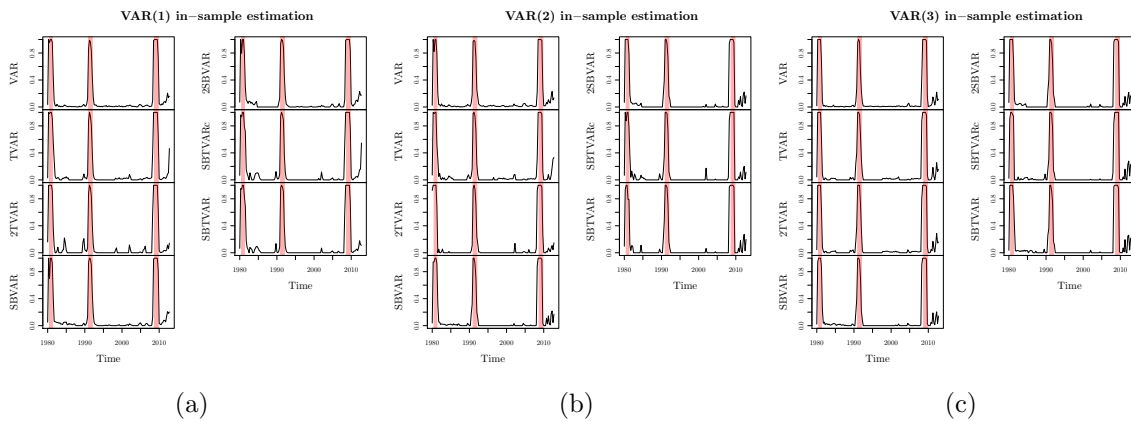


Figure 1: Recession in-sample estimation

Summarizing, models with non-linearity are able to model real growth and recession very well. 2SBVAR(2) with break point 1986q1 and 1991q3 is the best in-sample estimation among the models. This is an evidence of non-linear behaviour in the real growth-term spread relationship.

5.2. Out-of-sample forecast

In order to examine the strength of the prediction link in the real growth-term spread relationship, out-of-sample tests have been conducted. Three aspects of the out-of-sample tests are considered: sensibility to new information, performance of real growth forecasting and performance of recession forecasting.

5.2.1. Sensibility to New Information

In the out-of-sample test, thanks to the recursive estimation, one can record the dynamic relationship between the spread and output growth. Ideally, if the model is used

for forecast, then the parameters should be robust to introduction of new information. Models are estimated recursively to examine the robustness.

The in-sample period is chosen from 1980q1 to 2001q2, and out-of-sample period is 2001q3 to 2013q1. The model will be re-estimated each time a new point time join the sample. Therefore the in-sample period is actually the first estimation sample. 1980q1 to 2001q3 will be the second estimation sample, and keep re-estimating like this. The estimated parameters of TVAR, 2TVAR, SBVAR, 2SBVAR, SBTVARc, and SBTVAR are sorted into three sub-figures: delays, break points and thresholds.

Figure 2 to 4 shows the sorted recursively estimated parameters of models with one, two and three autoregressive order(s) respectively. For VAR(1) (see Figure 2) delays is quite stable with models with threshold only. Structural breaks are quite stable, one break is around late 1985 and gently increase on arrival of new information. And the second break and the break of SBTVAR(1) is 1991. There is only very short period of instability about the second break of 2SBVAR(1) during the financial crisis. Regarding thresholds, the model with threshold only show a very stable pattern, one is near 1 the other a little bit over 2. Figure 3 shows the parameters of model with 2 autoregressive orders. it is quite similar to the result from Figure 2. It worth mentioning that the structural breaks are stabler than VAR(1) models especially for the 2SBVAR. The instability happens in 2SBVAR(1) does not show up here. This confirms 2SBVAR(2) which is the best model from in-sample estimation enjoys the forecast ability's robustness as well. The two breaks estimated includes one in late 1985 and the other break around 1992. These breaks can be explained as I did in Section 5.1.1. This also indicate the big recession happened recently having very limited influence on the relationship modeled by 2SBVAR(2). The break points of SBVAR is increasing from 1986 to 1990 along with the new information coming. This behaviour of recursively estimated structural breaks and the stability of the two breaks indicate there are two and only two breaks in the sample. The delay parameters of VAR(3) (see Figure 4) show a fairly stable pattern in the recursively estimation. While the structural break and threshold parameters are quite unstable in models with 3 autoregressive orders. In summary, the 2SBVAR(2) shows its stable performance on arrival of new information, which confirms the robustness of the model.

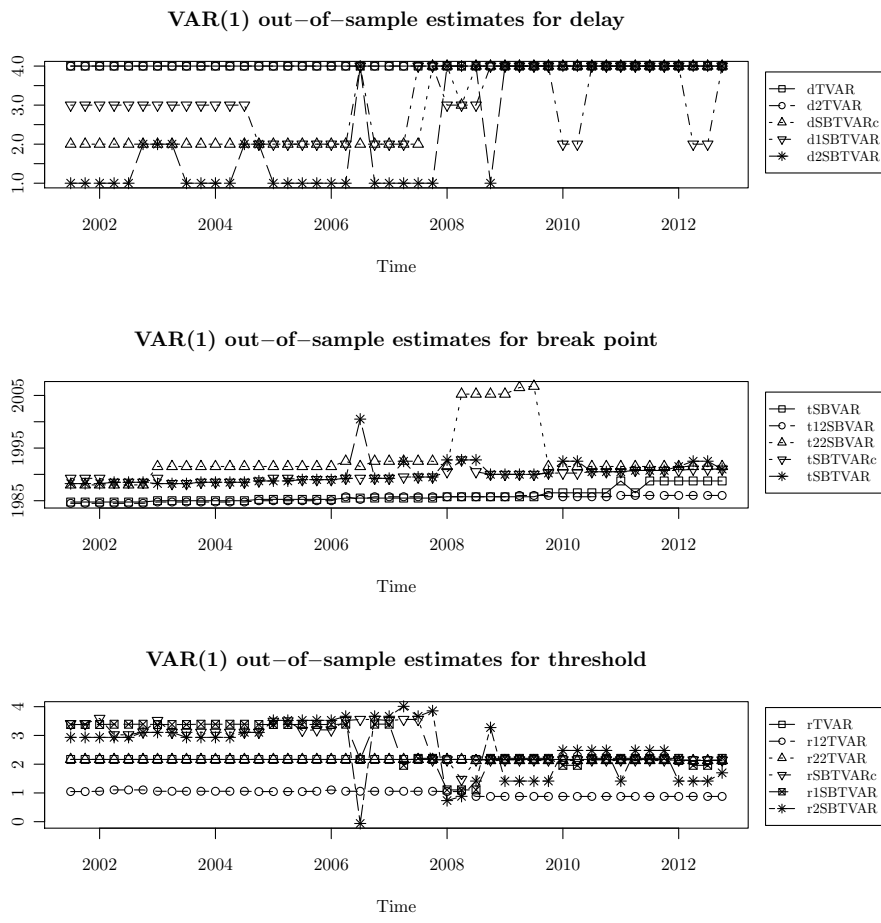
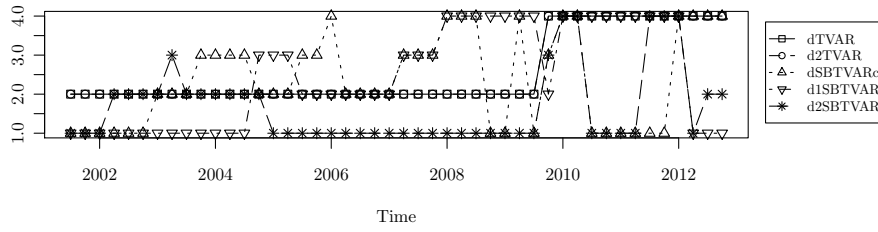
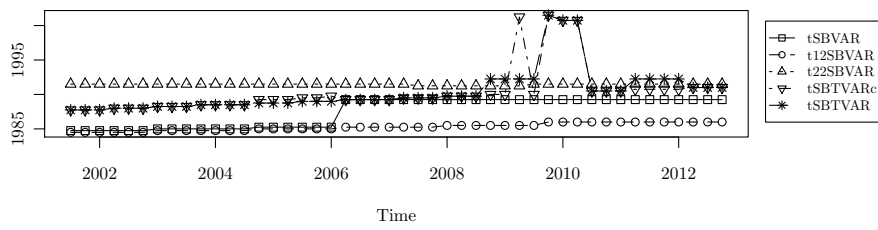


Figure 2: VAR(1) out-of-sample parameters

VAR(2) out-of-sample estimates for delay



VAR(2) out-of-sample estimates for break point



VAR(2) out-of-sample estimates for threshold

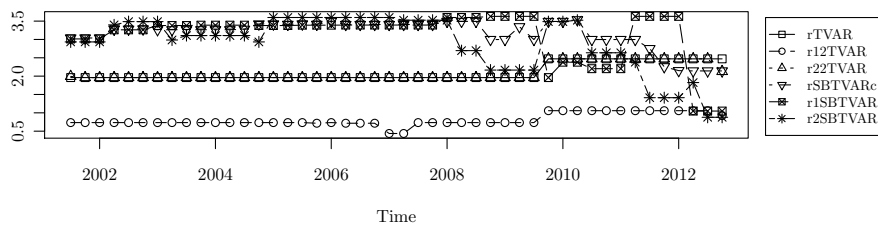


Figure 3: VAR(2) out-of-sample parameters

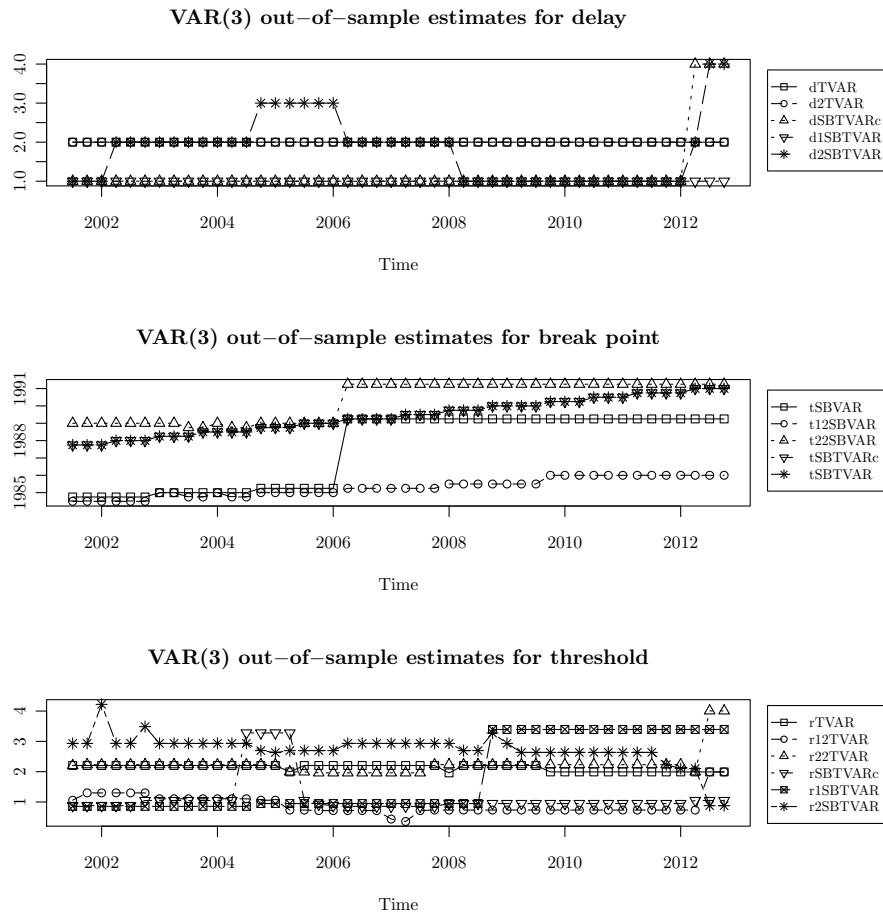


Figure 4: VAR(3) out-of-sample parameters

5.2.2. Real Growth Forecasting

Figure 5 to Figure 8 present the out-of-sample forecasting of all the model with forecasting horizon of 1 to 4 respectively.

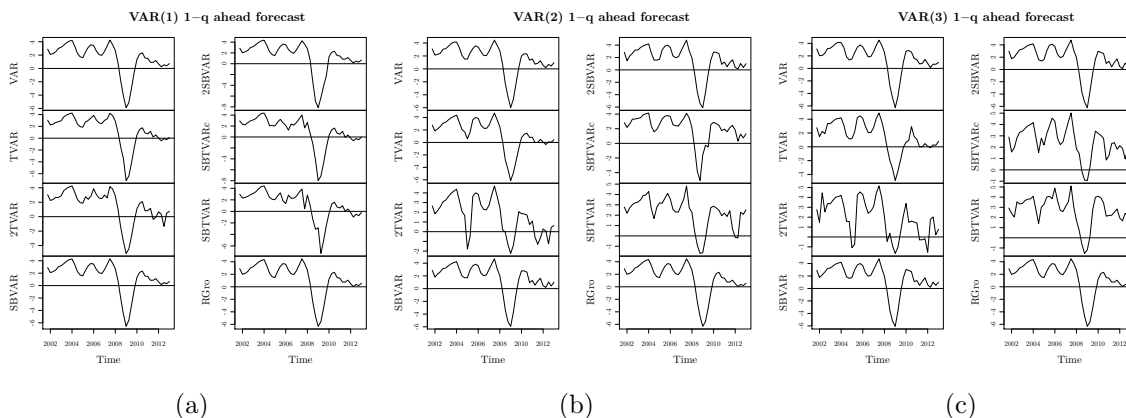


Figure 5: 1-quarter ahead real growth forecasts out-of-sample
 Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3-2013q1.

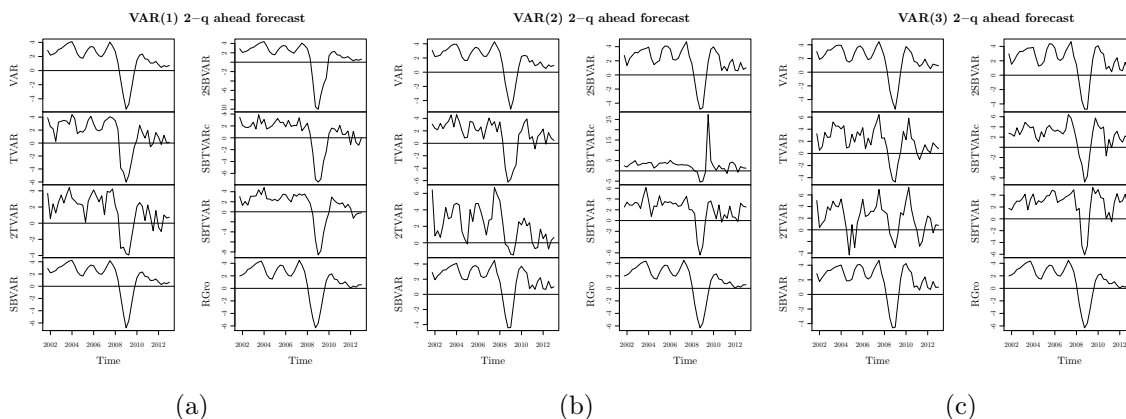


Figure 6: 2-quarter ahead real growth forecasts out-of-sample
 Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3-2013q1.

The last sub-sub-figure in each sub-figure is the real growth (RGro) for comparison. Across the figures, we can see that the growth forecasted by Threshold VARs are quite volatile especially with longer forecasting horizon. This is because, as mentioned in last section (Section 5.2.1), the threshold parameters are quite unstable when the model absorbs new information. It is also interesting to noted that, for SBTVARcs with 2 autoregressive order the predictions of real growth is quite abnormal in 2009 which pick around 25% (see Figure 6b, 7b and 8b). This could be the result of parsimonious problem when there a small amount of observations in one of the regimes. This issue might also exist in SBTVAR estimation, nevertheless the impact is not as big. linear models' (VARs) results are quite smooth in out-of-sample forecast as well as Structural Break

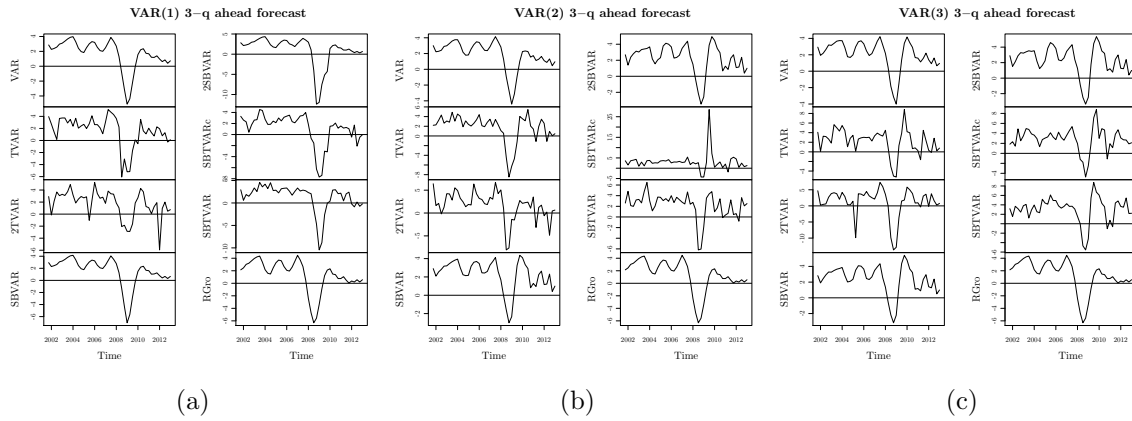


Figure 7: 3-quarter ahead real growth forecasts out-of-sample
 Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3-2013q1.

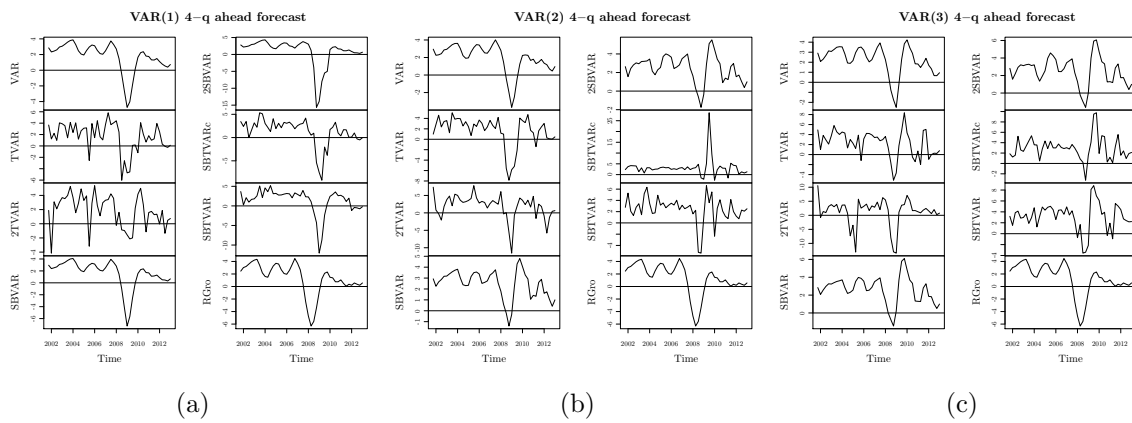


Figure 8: 4-quarter ahead real growth forecasts out-of-sample
 Note: in-sample period is 1979q1 to 2001q2; out-of-sample period is 2002q3-2013q1.

VARs. However, Structural break models does better in longer horizon forecasting. The performance of forecasting are examined and compared by Root Mean Square Error (RMSE) in the present paper. RMSE is a conventional tool to measure the efficiency of a forecast model in out-of-sample test. It is calculated as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^T (A_i - F_i)^2}{T}} \quad (4)$$

Where A is the actual value, and F is the forecast value. T is the out-of-sample period.

The results of forecasting performance comparison among the models are shown in Table 7. In terms of 1-quarter ahead forecasting, SBVAR(1) is outperform others with a RMSE score of 0.1193. VAR(3) enjoy the best performance in 2-quarter ahead forecasting. For 3-quarter and 4-quarter ahead forecasting, 2SBVAR(2) and SBVAR(2) get the lowest RMSE respectively. Generally speaking, Structural break VARs outperform the linear models (VARs) out-of-sample. It is worth mentioning that SBVAR makes a better forecasting model than 2SBVAR on average in the out-of-sample test. By summarizing the out-of-sample results, the best in-sample model is not necessarily the best out-of-sample. However, 2SBVAR(2) did beat others in 3-quarter ahead forecasting and is ranked second or third in 1-quarter 2-quarter and 4-quarter ahead forecasting. This means 2SBVAR(2) is a stable forecasting model for UK real growth forecast across different forecasting horizon.

Table 7: Comparison of out-of-sample forecast RMSE

Autoregressive order	1	2	3
	1-q ahead		
VAR	0.203719	0.330491	0.507225
TVAR	0.486583	0.401619	0.670783
2TVAR	0.602432	1.50678	1.713174
SBVAR	0.119349	0.713874	0.780364
2SBVAR	0.516948	0.786495	0.807101
SBTVAR _c	0.771787	1.394754	1.867266
SBTVAR	1.082038	1.638453	2.002732
	2-q ahead		
VAR	1.10935	1.040364	0.912622
TVAR	1.294086	1.182042	1.332358
2TVAR	1.358937	2.195417	2.154308
SBVAR	1.111953	0.98902	1.08046
2SBVAR	1.544483	0.994417	1.05831
SBTVAR _c	1.480936	4.460788	1.355875
SBTVAR	1.604809	1.482604	2.055407
	3-q ahead		
VAR	1.943813	1.849534	1.663777
TVAR	2.181866	1.944179	2.126007
2TVAR	2.237575	2.46949	3.087311
SBVAR	2.045546	1.678441	1.747401
2SBVAR	2.811096	1.658131	1.711788
SBTVAR _c	2.226587	4.874093	2.050893
SBTVAR	2.555783	1.94023	3.067268
	4-q ahead		
VAR	2.577424	2.458578	2.31837
TVAR	3.018232	2.7817	2.774163
2TVAR	2.82722	3.687261	3.749341
SBVAR	2.799693	2.281595	2.368796
2SBVAR	4.075094	2.300382	2.353885
SBTVAR _c	3.235476	5.209555	2.655144
SBTVAR	3.351514	2.642391	3.143473

5.2.3. Recession Forecasting

For this analysis, the in-sample and out-of-sample periods are as same as the studies conduct in the previous session. Figure 9a to 9c show the 3-quarter ahead recession forecasting abilities of estimation model using 1 autoregressive lag order to three autoregressive orders. Generally speaking, all the model can identify the most recent Financial Crisis. And they are all showing a increasing probably of recession after 2010. 2TVAR(3) falsely predicted a recession in 2005, and both 2TVAR(2) and 2TVAR(3) predicted another recession late 2012. It is interesting to note that models with threshold showing unstable of recession during the period 2008 to 2010. This result can be explained in

association with the issue Threshold VAR encountered in the previous session (Session 5.2.2). That is models with threshold(s) cannot digest new information very well in UK output growth-term spread relationship.

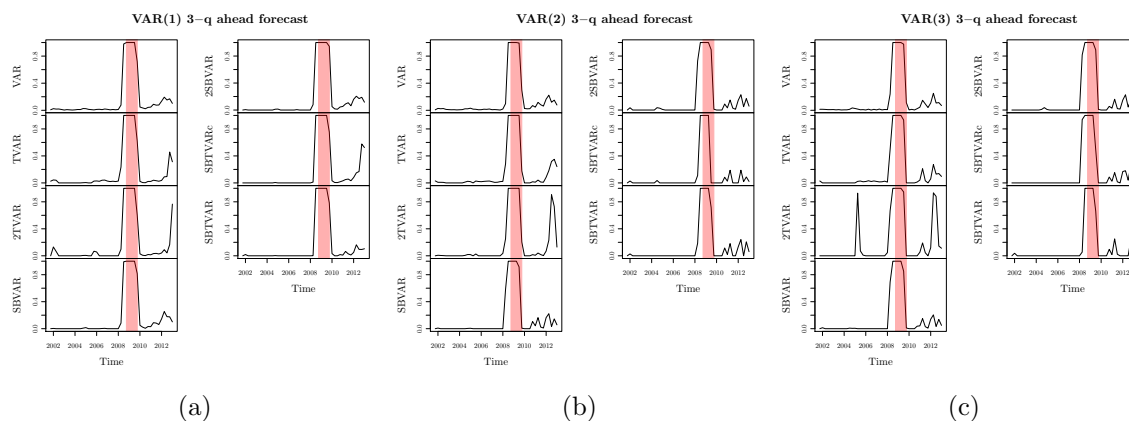


Figure 9: Recession out-of-sample forecast 3-q ahead

To sum up, in terms of recession forecasting, both linear and non-linear models work well except Threshold models.

6. Concluding Remarks

This paper examines the non-linear behaviour in output growth / recession-term spread relationship using UK data covering the last 34 years. The research conduct comparisons of VAR, TVAR, 2TVAR, SBVAR, 2SBVAR, SBTVAR_c, SBTVAR with 1 to 3 autoregressive order. The results suggest there are non-linearity in the relationship. And evidence show that the type of this non-linearity is structural break. By introducing structural break(s) into the model do improve the explainability of output growth- yield spread relationship as well as the prediction power of the model. 2SBVAR(2) is the tested best in-sample estimating model. And the model enjoy the robustness on arrival of new information. This also indicates the most recent financial crisis does not change the fundamental being of the relationship. SBVAR basically dominated the out-of-sample forecast. However, 2SBVAR is almost as good. In terms of recession predicting, models presented in the paper except Threshold models all give a fairly good performance.

Unavoidable that in this study, there are limitations about the comparison. For models with more regimes (SBTVAR_c and SBTVAR) in this limited sample size research, the disadvantage in the comparison is there might be very small regimes in the model. This will trigger the parsimony problem of the VAR estimations, especially for models with higher autoregressive orders, which could lead to abnormal forecasts.

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