Assortative Matching and Risk Sharing[*]

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Abstract

This paper analyzes sorting pattern of risk-sharing partnerships where agents are heterogenous in their income riskiness. When preference belongs to the class of HARA, household production in terms of monetary equivalence is perfectly transferable between spouses. Hence the characterization of stable match which minimizes social risk premium crucially depends on the interaction between risks in the household portfolio. In the multiplicative model where individuals are ranked by their holdings of a common risky stock, the convexity of household risk premium in joint risk size lead to negative assortative matching. In the additive model where individuals are ranked by their idiosyncratic risks in the Rothschild-Stiglitz sense, negative sorting is stable if any Rothschild-Stiglitz deterioration raises local risk aversion à la Ross.

Keywords Assortative matching, efficient risk sharing, transferable utility, systematic risk, idiosyncratic risk, risk vulnerability, Rothschild-Stiglitz increase in risk

JEL Classification C78, D31, D81, J12

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1 Introduction

The reflexive making of high modernity is characterized by significant increase in income volatility as well as an accelerated expansion of social insurance systems (Gottschalk and Moffitt, 1994; Moffitt and Gottschalk, 2002). Transitory exposures to such high risks are rarely reduced via formal consumption loans, as credit markets are typically absent or incomplete due to credit rationing or bad policy (Stiglitz and Weiss, 1981; Besley, 1994). Income diversification for effective risk-reduction appears limited for poorest wealth decile or for agricultural households whose farming largely depends on weather, crop prices and macroeconomic policies (Jalan and Ravallion, 1997). There is emerging consensus that efficient risk sharing across units such as extended families or villages is rejected (Altonji et al., 1992; Townsend, 1994). However, informal insurance arrangements through *inter vivos* transfers do occur within families, ethnic groups and professional networks. They fill in gaps in market failures and have intrinsic advantages in enforcement and monitoring (Kotlikoff and Spivak, 1981; Rosenzweig and Wolpin, 1985; Rosenzweig, 1988; Rosenzweig and Stark, 1989; Rosenzweig and Wolpin, 1994; Weiss, 1993; Coate and Ravallion, 1993; Grimard, 1997; Ligon et al., 2002; Fafchamps and Lund, 2003; Ayalew, 2003; Hess, 2004; Locay, 1990; Pollak, 1985).

The formation of marriage and kinship networks have implications on the allocation of risks at more aggregated levels. Marriage-related migrations occur when long-distance linkages are needed for diversifying risks due to local weather conditions. There is evidence in India that families arrange daughters married to men in sufficiently distant villages to reduce correlation in climatic and production shocks (Rosenzweig and Stark, 1989). It is also reported that the majority of relatives providing aids during drought conditions were located outside of the study villages (Caldwell et al., 1986). Moreover, a household's success in mitigating consumption shortfalls ex post via family ties importantly affects its ex ante evaluation of financial assets on the market or its willingness to bear risk through productive resources. This in turn has general equilibrium implications. When the nature of risk pattern is a primary concern in the marriage decision-making process, it is legitimate to ask what are the properties of such long-term yet informal risk-sharing mechanisms. More specifically, when individuals are crucially heterogenous in their risk characterizations.

There is a large body of game theoretic or experimental literatures which study the risk-sharing network formation as well as their consumption patterns (Weiss, 1993; Kocherlakota, 1996; Ligon et al., 2000, 2002; Taub and Chade, 2002; Genicot and Ray, 2003; Charness and Genicot, 2004; Bloch et al., 2005; Bramoulle and Kranton, 2005). Among the main insights of this flow of literatures is a fundamental complementarity arising from limited commitment or information asymmetry which eventually leads to social segmentation. However, as for the motivation of consumption insurance for spouses, dissimilar sorting of mates by risk attitudes will arise. Benefits from pooling within a household make the risk bearing of the male and the female substitutes for each other: very risk averse female is a demanding buyer for insurance and very risk tolerant male is a ready seller for it (Chiappori and Reny, 2006; Schulhofer-Wohl, 2006; Legros and Newman, 2007). On the other hand, sorting for insurance should not automatically be interpreted as the result of people's heterogenous preferences for risk. It may also, at least partially, be the result of their risk factors. This paper thus aims to examine the sorting patterns of risk-sharing partnerships where individuals are assumed to have identical preference but are differentiately affected by risks.

The problem is of considerable empirical importance. Income riskiness of individuals presumably are more easier to observe than their attitudes towards risk. In the expected utility framework, the shapes of utility functions are inferred from people's choices in laboratory or in managerial contexts. The potential difficulty with the assessment of a person's risk attitude is that different method and procedure often result in different classifications. Individual risk preferences have not been shown to be stable across different stimulus domains and situations. For example, the predictive power of investors' risk taking heavily depends on whether their risk attitudes are elicited in an investment related context (Slovic, 1964; MacCrimmon and Wehrung, 1986, 1990; Schoemaker, 1990, 1993; Weber and Milliman, 1997). On the other hand, income riskiness, which can be compared in an objective way, is an observable trait. One might expect to drive testable predictions concerning the role of risk-sharing in household institutions much more easily if individuals are ranked on the basis of their income riskiness.

Formally, we study a society comprised of two sides of individuals characterized by the risks they are endowed with. The sources of uncertainty considered in this paper include systematic risk which is associated with aggregate return and affects all agents simultaneously, and idiosyncratic risks which only affect a particular individual in the economy at a time. The former includes occurrences such as war, recession, inflation, fluctuating interest rates or other economy-wide climatic conditions. The latter includes risks on human capitals or other idiosyncratic shocks that rural and urban households in developing countries face. Since the asset markets do not fully span labor risks and access to proper and well-functioning financial markets is limited for developing economies, a considerable portion of wealth is nontradable. Both risks have been emphasized here as particularly relevant to marriage market where spouses mitigate individual consumption fluctuations by pooling incomes.

Individuals have the possibility to form marriages to mutualize risks. In competing for potential partners, the decision to form and maintain a particular marriage depends on the whole range of opportunities and not only on the merits of specific risk-sharing partnership. Becker's (1973, 1974, 1981) landmark optimal sorting models provide the foundation for analyzing two-sided matching market. When household production is perfectly transferable between spouses, the criteria for stable match is to maximize total output over all possible marriages, as it is then impossible for both spouses to gain from rematch or remain single. With respect to risk-sharing problem, it is well known that if the preference belongs to the class of Harmonic Absolute Risk Aversion (HARA), i.e., agents' utility functions are all exponential, all logarithmic or all common power, the Pareto frontier in the monetary-equivalent space is a straight line. This restriction guarantees the matching game to permit transferable expected utility representation. Hence, with proper choice of expected utility transformation, the key dimension of match output is the joint cost of household risk premium. Becker's program on competitive equilibrium matching translates into minimizing risk premiums summed over all households. It crucially depends on the interaction between risks in every household's portfolio.

In the systematic risk model where individuals are ranked by their holdings of a common risky stock, household risk premium is convex in the numbers of stock in the household's portfolio. That is, there is aggravating effect with intrahousehold equality of income riskiness. Hence negative sorting of mates is socially preferable and stable. In the idiosyncratic risk model where individuals are ranked by their independent riskiness in the sense of Rothschild-Stiglitz (1970), HARA preference is not sufficient to generate clear result. Additional restrictions must be imposed on the preference regardless of distribution of risk. The concept of risk vulnerability, coined by Gollier and Pratt (1996) turns out to be particularly relevant. Risk vulnerability guarantees that the deterioration in a risk taking the form of adding an independent pure white noise increases the cost of the existing risk. Under HARA utility, Decreasing Absolute

Risk Aversion (DARA) is necessary and sufficient for risk vulnerability.

As an illustration, consider two agents endowed with risk-free wealth and two with risky wealth. In the case of negative sorting, the social risk premium is the sum of two household risk premiums, each related with one source of risk. In the case of positive sorting, the social risk premium is the risk premium of only one household, i.e., the one pools two risky income. We thus aim to compare the sum of risk premiums and the premium of the summed risks. For the positive sorting, on one hand, the presence of a mean-zero background risk decreases the monetary value of the existing risky prospect; on the other hand, a reduction in monetary equivalence raises the premium to the mean-zero risk itself. In this sense, there is aggravating effect with multiple risk-bearing. Hence social risk premium is minimized when sorting is negative. As a result, negative sorting is stable in competitive market. In general, negative assortative matching (NAM) on spouses' income riskiness à la Rothschild-Stiglitz is stable if any deterioration in risk raises local risk aversion à la Ross.

This paper is part of the substantial body of literature on the economics of family formation. Becker's (1973, 1974, 1981) landmark optimal sorting models provide foundation for analyzing the assignment of partners and the sharing of gains from marriage. On the two-sided market where individuals within each population differ in one-dimensional characteristics such as education or wage, competition leads to first best marital sorting which maximizes total output from all marriages. When household production are perfectly transferable between spouses, increasing difference in spouses' traits is sufficient for positive assortative matching to arise. As for the risk-sharing partnership formation, with proper choice of expected utility transformation, Becker's program on stable match translates into minimizing social risk premium.

Our analysis is closely related to Chiappori and Reny (2006) and Schulhofer-Wohl (2006) where the role of risk-sharing in household composition is discussed with a minimalist assumption of Pareto-efficiency of household outcome. Individuals on the marriage market seek to find appropriate risk-sharing partners based on their attitudes towards risk in the Arrow-Pratt sense. Their results support the intra-household heterogeneity in risk aversion and indicate its importance in econometric practice with consumption data. The current paper formulates agents' heterogeneity as on their income riskiness, where agents are either endowed with different numbers of a risky stock or their idiosyncratic risks are ranked in the Rothschild-Stiglitz sense.

The paper by Jaramillo, Kempf and Moizeau (2009) is also relevant as it is concerned with the formation of risk-sharing coalitions where individuals differ with respect to their risky exposure. The insurance scheme considered in their work is limited to equal sharing regardless of agents' initial incomes, while in our model there is no barrier to efficient insurance within the household.

The predictions regarding heterogeneity in risk on the matching pattern is important. They connect the theory of matching with the most important preference concept regarding decision making under multiple sources of risks. When individuals' pre-existing risks are perfectly correlated, pooling multiple risks are self-aggravating in the sense that joint risk premium is a convex function of the joint size of the risks (Eeckhoudt and Gollier, 2001). When individuals' pre-existing risks are independent, risk vulnerability guarantees that adding an exogenous unfair background risk raises risk aversion with respect to any other independent risk (Gollier and Pratt, 1996; Eeckhoudt, Gollier and Schlesinger (1996)). The current paper shows that when preference belongs to the HARA class, convex cost of risk and risk vulnerability are central to the explanation of the equilibrium sorting pattern.

The rest of the article is organized as follows. We first study a stylized risk-sharing matching game in Section 2. The characterization of stable match is laid out. In Section 3, we visit risk sharing problem which has a transferable expected utility representation. It is shown that if preference belongs to the HARA class, the matching surplus in terms of monetary equivalent is free of intra-household distribution. In Section 4, we examine the situation where income risks are perfectly correlated. We lend insights from several preliminary examples and then we derive the general result. In Section 5, we show the basic intuition with idiosyncratic risks in a 2×2 case where two agents are endowed with certain income and two with risky income. We then study the general case where income riskiness is ranked in the Rothschild-Stiglitz sense. We conclude in Section 6.

2 Risk-Sharing Matching Game

Consider a stylized marriage market with N males $\{i = 1, ..., N\}$ on one side and N females $\{j = 1, ..., N\}$ on the other side. Each agent is endowed with a fixed initial income, some units of a common risky stock and also faces an uninsurable idiosyncratic risk. Denote $\tilde{w}_i = w_0 + k_i\tilde{x} + \tilde{\varepsilon}_i$ and $\tilde{w}_j = w_0 + k_j\tilde{x} + \tilde{\varepsilon}_j$ as i's and

j's income respectively, where w_0 is the certain component, $\tilde{x}$ is the gross aggregate return per unit of risky stock, k_i and k_j, $i, j \in \{1, ..., N\}$ denote the number of units owned by i and j, and $\tilde{\varepsilon}_i$s and $\tilde{\varepsilon}_j$s are independently distributed for $\forall i, j \in \{1, ..., N\}$. All agents are expected utility maximizers with respect to homogeneous probabilistic belief, and identically risk averse with vNM utility function $u(c) : [0, \infty) \to \mathbb{R}$ which is bounded and continuously differentiable in consumption c, $u'(c) > 0$, $u''(c) < 0$ for all $c \in (0, \infty)$, and $u(0) = 0$.

Agents can form marriages to share risks. At period 0 each agent voluntarily matches with a mate from the opposite side. Each couple (i, j) will commit to rules for sharing their income in each state of the world. At period 1, the value of all shocks are realized, and agents consume according to the prior sharing rules. We assume away any search or coordination frictions at the moment, neither is there limited commitment or asymmetric information. Denote $\tilde{z}_{ij} =_d 2w_0 + (k_i + k_j)\tilde{x} + (\tilde{\varepsilon}_i + \tilde{\varepsilon}_j)$ as the household income received by the couple (i, j). Associated with a marriage are divisions ($z_{ij} - c_{ij}$, c_{ij}) prior to the realization of shocks that specify individual consumptions under each realization of $\tilde{z}_{ij}$ to husband i and wife j respectively. Under this agreement, husband i's dowry is $Eu(\tilde{z}_{ij} - c_{ij}(\tilde{z}_{ij}))$ and wife j's bride price is $Eu(c_{ij}(\tilde{z}_{ij}))$.

Assume that risk is shared within each couple in a Pareto-efficient way, a situation where no agent's expected utility can be increased without decreasing her partner's. This in turn requires that the allocation of consumption satisfies the principle of mutuality (Wilson, 1968). That is, individual j's state-contingent consumption level c_{ij} is a deterministic function, called the ***risk sharing rules***, that map each realized value of z_{ij} to an individual consumption level for j. The set of Pareto-optimal allocations, or so-called Pareto efficient frontier, is give by: for some scalar $\lambda \in \mathbb{R}_{++}$ and all z_{ij}, risk sharing rules $c_{ij}(z_{ij})$ solve the following maximization problem

$$\max_{c_{ij}} \{Eu(\tilde{z}_{ij} - c_{ij}(\tilde{z}_{ij})) + \lambda Eu(c_{ij}(\tilde{z}_{ij}))\} \tag{1}$$

Definition 1 *A **matching correspondence** is an assignment of men to women and a **stable match** specifies a matching correspondence and associated sharing rules for matched agents which are immune to coalitional deviation; that is, there doesn't exist a matched man and a matched woman who prefer to form a couple other than their current match.*

Assume k_is, k_js, $\tilde{\varepsilon}_i$s and $\tilde{\varepsilon}_j$s strictly differ within each side of the population, further assume that marginal utility of consumption are bounded at autarky. Existence of stable matches has been established by Legros and Newman (2007: 9.1). When the traits of husbands and wives correspond to wealth and education, Becker (1973) proposed the concept of positive/negative assortative matching (PAM/NAM), which refers to a positive/negative correlation in sorting between the traits of husbands and wives, i.e., matching of likes/unlikes. In this paper, the traits to be considered are the two potential partners' income riskiness, either as the unit holdings of the common stock, k, or as the ranking of $\tilde{\varepsilon}$ in the Rothschild-Stiglitz sense. Consequently, in the case of positive/negative assortative matching, the most risky man is matched with the most/least risky woman, the second most risky man is matched with the second most/least risky woman. The formal definition of equilibrium matching pattern is stated as follows:

Definition 2 *Stable match satisfies* ***positive (negative) assortative*** *iff for any i, i' and j, j' such that i is matched with j and i' is matched with j' in equilibrium, we have*

$$i' \geq i \Longleftrightarrow j' \geq (\leq) j$$

3 Stable Match and Social Risk Premium

This paper will examine a special class of preferences under which the risk-sharing matching problem admits transferable expected utility representation. Transferable Utility (TU) is assumed in many cooperative games, where players are allowed to form binding agreements, act collectively to obtain maximum total output and split the gain costlessly. Full transferability admits some choices of utility function representing agents' preferences such that the generated utility possibility frontier is a simplex. The definition of transferable expected utility representation is stated as follows:

Definition 3 *The risk-sharing matching game has* ***transferable expected utility representation*** *iff the sum of spouses' certainty equivalent consumption levels is independent of intra-household allocation coefficient λ,i.e., it equals some constant C_{ij} for all efficient risk sharing rules $c_{ij}(z_{ij})$ adopted by potential* *partners* $(i,\, j)$*:*

$$u^{-1}[Eu(c_{ij}(\tilde{z}_{ij}))] + u^{-1}[Eu(\tilde{z}_{ij} - c_{ij}(\tilde{z}_{ij}))] = \ C_{ij}.$$

A central implication of the above TU representation definition is that, the household output in terms of certainty equivalent, C_{ij}, produced by the potential couple (i, j) depends only on characteristics of members' joint income distribution $\tilde{z}_{ij}$, and is free of divisions within marriage. This output measure allows individuals to compare the gains from marriage she may acquire with various potential mates. As we will see later, the condition for stable match is simply to maximize the social output $\sum_{i,j} C_{ij}$ summed over the entire population, which is equivalent to minimizing social risk premium, i.e., the monetary evaluation of social cost of risk. Denote the ***Household Risk Premium*** for the risk-sharing matching game as $\pi_{ij} = E\tilde{z}_{ij} - C_{ij}$ and associated ***Social Risk Premium*** as $\sum_{i,j} \pi_{ij}$.

The existence of TU representation is subject to certain regularity conditions. With respect to risk-sharing problem, it is well known that preference belongs to the class of Harmonic Absolute Risk Aversion (HARA) if and only if the Pareto frontier in the monetary-equivalent space is a line with constant slope. This suggests that HARA preference is necessary and sufficient for transferable expected utility representation.

Definition 4 *Preference belongs to the class of* ***Harmonic Absolute Risk Aversion (HARA)*** *iff absolute risk tolerance is a linear function of consumption:*

$$T\left(c\right) = \frac{1}{\gamma}c + \frac{1}{\alpha} \tag{2}$$

where risk tolerance $T(c) = -u'(c)/u''(c) > 0$ *is the reciprocal of Arrow-Pratt measure of absolute risk aversion. In particular, preference exhibits* ***Decreasing/Increasing Absolute Risk Aversion (DARA/IARA)*** *if* $\gamma > 0$ *(*$\gamma < 0$*),* ***Constant Absolute Risk Aversion (CARA)*** *if* $\gamma \to \infty$*,* ***Constant Relative Risk Aversion (CRRA)*** *if* $\alpha \to \infty$, *and* ***risk neutral*** *if* $\gamma \to 0$.

Lemma 1 *The risk-sharing matching game has transferable expected utility representation iff preference belongs to the HARA class.*

Proof. This result is well known. A recent proof can be found in Mazzocco (2004) and Schulhofer-Wohl (2006) combined. We provide an illustration of the sufficiency here. Solving for $u'(c) = D_1 T(c)^{-\gamma}$, where D_1 is a constant, from (2) and combining with the F.O.C. of Pareto optimization (1), yields:

$$c_{ij}^*(\lambda) = \frac{\frac{\gamma}{\alpha}\left(1 - \lambda^{\frac{-1}{\gamma}}\right) + z_{ij}}{1 + \lambda^{\frac{-1}{\gamma}}}$$

Substituting into the expression of risk tolerance (2), yields:

$$T\left(c_{ij}^*\right) = \frac{\frac{\gamma}{\alpha} + z_{ij}}{\gamma\left(1 + \lambda^{\frac{-1}{\gamma}}\right)};\ T\left(z_{ij} - c_{ij}^*\right) = \frac{\lambda^{\frac{-1}{\gamma}}\left(z_{ij} + \frac{\gamma}{\alpha}\right)}{\gamma\left(1 + \lambda^{\frac{-1}{\gamma}}\right)} \tag{3}$$

Since $C_{ij}(\lambda) = u^{-1}(Eu(c_{ij}^*(\lambda)) + u^{-1}(Eu(z_{ij} - c_{ij}^*(\lambda)))$, which after taking derivative w.r.t. λ and combining with (2) yields:

$$\frac{\partial C_{ij}}{\partial \lambda} = \gamma \frac{\partial}{\partial \lambda}\left(T\left(u^{-1}(Eu(c_{ij}^*(\lambda))\right) + T\left(u^{-1}(Eu(z_{ij} - c_{ij}^*(\lambda)))\right)\right) \tag{4}$$

Then, solving for $u(c)$ from (2), and combining with (3), we have $T\left(u^{-1}(Eu(c_{ij}^*(\lambda))\right) = \left(ET\left(c_{ij}^*(\lambda)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$ and $T\left(u^{-1}(Eu(z_{ij} - c_{ij}^*(\lambda)))\right) = \left(ET\left(z_{ij} - c_{ij}^*(\lambda)\right)^{1-\gamma}\right)^{\frac{1}{1-\gamma}}$. Inserting into (4), we can easily derive $\partial C_{ij}/\partial\lambda = 0$. ■

Becker's (1973) seminal paper provided foundation for analyzing competitive assignments of partners. On the two-sided matching market, suppose males and females are ranked by their types θ_i^m and θ_j^f. Under our specification, the certainty equivalent frontier for $(i,\, j)$ is the set

$$\{(C_i, C_j) \geq 0 \mid C_i + C_j = C_{ij} = C(\theta_i^m, \theta_j^f)\}$$

Notice that $C_i + C_j$ only depends on spouses' types. Following Becker (1973), we know that the stable match will be negative (positive) assortative on spouses' types θ_i and θ_j if C_{ij} is submodular (supermodular) in (i, j), or equivalently,

$$C(\theta_2^m, \theta_2^f) - C(\theta_2^m, \theta_1^f) < (>) C(\theta_1^m, \theta_2^f) - C(\theta_1^m, \theta_1^f). \tag{5}$$

Intuitively, the left-hand side of (5) is the maximum gain that male 2 can get by switching from female 1 to female 2, while the right-hand side is the counterpart for male 1. When submodular condition holds, male 1 can always outbid male 2 for exchanging female 1 with female 2 and still get no less for himself. Hence, stable match is negative. Rewriting the submodular (supermodular) condition yields

$$C(\theta_2^m,\theta_2^f)+C(\theta_1^m,\theta_1^f)<(>)\,C(\theta_1^m,\theta_2^f)+C(\theta_2^m,\theta_1^f), \tag{6}$$

which suggests a negative (positive) interaction in the production of marital output. The above condition suggests that people will match in such a way that the social certainty equivalent, defined as the sum of the household certainty equivalent, is maximized. Or in other words, the social risk premium is minimized.

If the types θ_i^m and θ_j^f are real numbers, then the differential condition of (5) is

$$\frac{\partial^2 C(\theta_i^m,\theta_j^f)}{\partial\theta_i^m\partial\theta_j^f}\geq 0\ (\leq 0). \tag{7}$$

The above differential condition corresponds to Topkis (1978)'s characterization of supermodularity (submodularity).

In the TU game with identical HARA preference, the attitude toward the risk born by a couple (i,j) is irrelevant of the choice of λ but rather given by a household representative agent with utility function $v\left(c\right)$. Choosing $\lambda=1$, we know $v\left(c\right)=2u\left(\frac{c}{2}\right)$ and risk tolerance is

$$\begin{aligned}T_v\left(c\right) &= T\left(c\right)+\frac{1}{\alpha}\\ &= \left(\frac{1}{\gamma}c+\frac{1}{\alpha}\right)+\frac{1}{\alpha}.\end{aligned}$$

Obviously v belongs to the class of HARA as well. When $\alpha\rightarrow+\infty$, i.e., preference exhibits CRRA, the household representative agent has the same risk aversion as the individual: $T_v\left(c\right)=T\left(c\right)$; when $\gamma\rightarrow+\infty$, i.e., preference exhibits CARA, the risk tolerance of the household representative agent doubles that of an individual: $T_v\left(c\right)=2T\left(c\right)$. In an abuse of notation, we denote π and C the risk premium and certainty equivalent functions as with preference v for the rest of the paper. And the related household risk premium is $\pi_{ij}=E\tilde{z}_{ij}-C_{ij}$ where $C_{ij}=v^{-1}[Ev(\tilde{z}_{ij})]$.The Lemma immediately follows.

Lemma 2 *If preference belongs to the HARA class, the stable match of the risk-sharing matching game will be positive (negative) assortative on spouses' income riskiness if household risk premium π_{ij} is submodular (supermodular) in (i,j).*

4 Sorting over Systematic Risky Exposures

In a systematic risk model, individuals are ranked by their unit holdings of perfectly correlated risks ($\tilde{\varepsilon}_i = \tilde{\varepsilon}_j = 0$). As a result of market competition, stable match guarantees the minimization of social cost of risk. Denote $k_{ij} \triangleq k_i + k_j$. Notice that $\pi_{ij} = E\tilde{z}_{ij} - v^{-1}[Ev(\tilde{z}_{ij})]$ and $\tilde{z}_{ij} = 2w_0 + k_{ij}\tilde{x}$, we have π_{ij} as a function of k_{ij} only. By Lemma 2 and differential condition (7), stable match will be positive (negative) assortative on risk sizes k_i, k_j if

$$\frac{\partial^2 \pi_{ij}}{\partial k_{ij}^2} \leq 0 (\geq 0)$$

i.e., household risk premium concave (convex) in the size of household risky exposure. As suggested by Eeckhoudt and Gollier (2001), multiplicative risk is self-aggravating in the sense that the cost curve of risk $\pi(k)$ is convex in the unit holdings of such risk k. Thus, we have the following proposition.

Proposition 1 *If preference belongs to the HARA class, the stable match of the risk-sharing matching game is negative assortative on spouses' sizes of systematic risky exposure. In particular, matching is arbitrary iff preference exhibits CRRA and initial fixed income is zero.*

Proof. Firstly $\pi_{ij} = E\tilde{z}_{ij} - v^{-1}[Ev(2w_0 + k_{ij}\tilde{x})]$ and thus

$$\frac{\partial^2 \pi_{ij}}{\partial k_{ij}^2} = -\frac{v'(v^{-1}(Ev(2w_0 + k_{ij}\tilde{x})))E\left(v''(2w_0 + k_{ij}\tilde{x})\tilde{x}^2\right) - \left(Ev'(2w_0 + k_{ij}\tilde{x})\right)^2 \frac{v''(v^{-1}(Ev(2w_0+k_{ij}\tilde{x})))}{v'(v^{-1}(Ev(2w_0+k_{ij}\tilde{x})))}}{[v'(v^{-1}(Ev(2w_0 + k_{ij}\tilde{x})))]^2},$$

which yields that $\frac{\partial^2 \pi_{ij}}{\partial k_{ij}^2} \geq 0$ iff

$$-\frac{v''(v^{-1}(Ev(2w_0 + k_{ij}\tilde{x})))}{[v'(v^{-1}(Ev(2w_0 + k_{ij}\tilde{x})))]^2} \leq -\frac{E(v''(2w_0 + k_{ij}\tilde{x})\tilde{x}^2)}{[E(v'(2w_0 + k_{ij}\tilde{x})\tilde{x})]^2} \tag{8}$$

Solving $v(c)$ from (2) and substituting into (8) yield that $\frac{\partial^2 \pi_{ij}}{\partial k_{ij}^2} \geq 0$ iff

$$[E(T_v(2w_0 + k_{ij}\tilde{x})^{-\gamma}\tilde{x})]^2 \leq E(T_v(2w_0 + k_{ij}\tilde{x})^{-(1+\gamma)}\tilde{x}^2)ET_v(2w_0 + k_{ij}\tilde{x})^{1-\gamma}$$

which holds as a direct application of Cauchy-Schwarz inequality. In particular, equality holds iff there exists a constant δ, such that $T\left(k_{ij}\tilde{x}\right)^{-\frac{(1+\gamma)}{2}}\tilde{x}=\delta T\left(k_{ij}\tilde{x}\right)^{\frac{1-\gamma}{2}}$, which is the case iff $\alpha\to\infty$ and $w_0=0$. ■

We illustrate the above result with the following examples.

Example 1a (*CARA utility with perfectly correlated normal incomes*). Suppose that there are two males m_1, m_2 and two females f_1, f_2. All agents have identical exponential utility $u(c)=-\frac{1}{\alpha}e^{-\alpha c}$ which is unique in exhibiting CARA. Agents are endowed with $w_0+k_i^m\tilde{x}$, $i=1,2$ and $w_0+k_j^f\tilde{x}$, $j=1,2$ where $\tilde{x}\sim N(\mu,\sigma^2)$. Firstly notice that $\tilde{z}_{ij}\sim N(2w_0+k_{ij}\mu, k_{ij}^2\sigma^2)$. Secondly, the representative agent has constant coefficient of absolute risk aversion $\frac{\alpha}{2}$. Under CARA and normal income, the Arrow-Pratt approximation is exact: $\pi_{ij}=\frac{1}{4}\alpha\sigma^2k_{ij}^2$. It follows immediately that $\frac{\partial^2\pi_{ij}}{\partial k_{ij}^2}>0$ and we conclude that stable match is strictly negative assortative.

Example 1b (*CRRA utility*). Suppose alternatively that all agents have power utility $u(c)=\frac{1}{1-\gamma}c^{1-\gamma}$. Under this particular CRRA assumption, the representative agent has exactly the same utility function as any individual: $v\left(c\right)=u\left(c\right)$. When $w_0=0$,by noticing that $u^{-1}\left(Eu\left(k_{ij}\tilde{x}\right)\right)=k_{ij}u^{-1}\left(Eu\left(\tilde{x}\right)\right)$, we immediately have $\frac{\partial^2\pi_{ij}}{\partial k_{ij}^2}=0$,and hence stable match is arbitrary. When $w_0>0$,

$$\frac{\partial u^{-1}\left(Eu\left(\tilde{z}_{ij}\right)\right)}{\partial k_{ij}^2}=\gamma\left[E\left(\tilde{z}_{ij}\right)^{1-\gamma}\right]^{\frac{2\gamma-1}{1-\gamma}}\left\{\left[E\left(\left(\tilde{z}_{ij}\right)^{-\gamma}\tilde{x}\right)\right]^2-E\left(\tilde{z}_{ij}\right)^{1-\gamma}E[(\tilde{z}_{ij})^{-\gamma-1}\tilde{x}^2]\right\}<0,$$

where the last inequality is due to Cauchy-Schwarz inequality. Hence the stable match is strictly negative assortative.

5 Sorting over Idiosyncratic Risks

In an idiosyncratic risk model where individuals are ranked by their independent risks in the Rothschild-Stiglitz (1970) sense ($k_i=k_j=0$), HARA preference is not sufficient to generate clear result and additional restrictions must be imposed on the preference for arbitrary distribution of risky income. By Lemma 2, the condition for NAM is that π_{ij} being supermodular in (i,j), or equivalently, $v^{-1}[Ev(\tilde{z}_{ij})]$ being submodular in (i,j).Before proceeding, we borrow some insights from the following examples.

5.1 Some Preliminary Examples

Example 2a (*CARA utility*). Suppose there are two males m_1, m_2 endowed with $w_0 + \tilde{\varepsilon}_i^m$, $i = 1, 2$ and two females f_1, f_2 endowed with $w_0 + \tilde{\varepsilon}_j^f$, $j = 1, 2$, where $\tilde{\varepsilon}_i^m$ and $\tilde{\varepsilon}_f^f$ are independent. Since all agents have identical CARA utility, π_{ij} is additive over $(\tilde{\varepsilon}_i, \tilde{\varepsilon}_j)$: $\pi_{ij} = \pi\left(\tilde{\varepsilon}_i\right) + \pi\left(\tilde{\varepsilon}_j\right)$. Therefore, π_{ij} is both (but not strictly) supermodular and submodular in (i, j). In contrast to systematic risk-sharing matching result, CARA preference leads to arbitrary matching.

Example 2b (*Quadratic utility with one certain income and one risky income*). Suppose instead that all agents have quadratic utility $u(c) = c - \frac{c^2}{2}$ which exhibits IARA. For the sake of simplicity, we assume that m_1 and f_1 are endowed with riskless income w_0, m_2 and f_2 are endowed with risky income $w_0 + \tilde{\varepsilon}^m$ and $w_0 + \tilde{\varepsilon}^f$, where $\tilde{\varepsilon}^m$ and $\tilde{\varepsilon}^f$ are distributed independently with zero mean and variance σ_m^2 and σ_f^2. Notice that for quadratic utility, the mean-variance approach is exact. And certainty equivalent function $u^{-1}(u) = 1 - \sqrt{1 - 2u}$. We have household risk premium $\pi_{ij} = w_0 - (1 - \sqrt{(1 - w_0)^2 + Var(\tilde{z}_{ij})})$. We can easily show that $\pi_{ij} = \pi(Var(\tilde{z}_{ij}))$ is concave in $Var(\tilde{z}_{ij})$, under which $\pi(0) + \pi(\sigma_m^2 + \sigma_f^2) \leq \pi(\sigma_m^2) + \pi(\sigma_f^2)$. It follows from (6) that stable match satisfies PAM. In contrast to results with systematic risk, PAM may arise if preference exhibits IARA.

Example 2c (*Logarithm utility with independent discrete incomes*). Suppose alternatively that all agents have logarithm utility function $u\left(c\right) = \ln c$ which exhibit DARA. m_1 and f_1 are endowed with certain income $w_0 = 3$, m_2 and f_2 are endowed with risky income $3 + \tilde{\varepsilon}^m$ and $3 + \tilde{\varepsilon}^f$. $\tilde{\varepsilon}^m$ and $\tilde{\varepsilon}^f$ are i.i.d., and $\Pr\left(\tilde{\varepsilon}^m = 1\right) = \Pr\left(\tilde{\varepsilon}^m = -1\right) = \frac{1}{2}$. Simple calculation gives $\pi(\tilde{\varepsilon}^m, 0) = \pi(0, \tilde{\varepsilon}^f) = 0.17157$ and $\pi(\tilde{\varepsilon}^m, \tilde{\varepsilon}^f) = 0.40998$. Obviously, $\pi(\tilde{\varepsilon}^m, \tilde{\varepsilon}^f) > \pi(\tilde{\varepsilon}^m, 0) + \pi(0, \tilde{\varepsilon}^f)$ and hence the stable match satisfies NAM.

The above examples suggest that all sorting patterns are possible with idiosyncratic risks. Moreover, DARA might play an important role for NAM to arise.

5.2 An Illustration with Degenerated Income

In this section, we investigate the 2×2 case where m_1 and f_1 are endowed with certain income $w_1^m = w_1^f = w_0$ (i.e., $\tilde{\varepsilon}_1^m = \tilde{\varepsilon}_1^f = 0$), m_2 and f_2 are endowed with risky income $\tilde{w}_2^m = w_0 + \tilde{\varepsilon}^m$ and $\tilde{w}_2^f = w_0 + \tilde{\varepsilon}^f$. In order to focus on the effect of riskiness, assume $E\tilde{\varepsilon}^m = E\tilde{\varepsilon}^f = 0$.With a slight abuse of notation, we introduce the following definition.

Definition 6 *(Gollier and Pratt, 1996) The **generalized risk premium** $\Pi_{\tilde{\varepsilon}}(\tilde{x}, w)$ of risk $\tilde{x}$ in the presence of initial wealth w and background risk $\tilde{\varepsilon}$, is the price that the representative agent would be willing to pay to avoid the risk $\tilde{x}$ at an uncertain position $w + \tilde{\varepsilon}$: $Ev(w + \tilde{\varepsilon} + \tilde{x}) = Ev(w + \tilde{\varepsilon} - \Pi_{\varepsilon}(\tilde{x}, w))$.The **conventional risk premium** $\pi(\tilde{x}, w)$, defined by $Ev(w + \tilde{x}) = Ev(w - \pi(\tilde{x}, w))$, is a special case of the generalized risk premium with no background risk: $\tilde{\varepsilon} \equiv 0$.*

Now we derive an equivalence which will be useful later. For risks $\tilde{x}$, $\tilde{y}$ and $\tilde{z}$, by the above definition, we have

$$\begin{aligned} Eu(w - \Pi_{\tilde{x}}(\tilde{y} + \tilde{z}, w) + \tilde{x}) &= Eu(w + \tilde{x} + \tilde{y} + \tilde{z}) \\ &= Eu(w - \Pi_{\tilde{x}+\tilde{y}}(\tilde{z}, w) + \tilde{x} + \tilde{y}) \\ &= Eu(w - \Pi_{\tilde{x}+\tilde{y}}(\tilde{z}, w) - \Pi_{\tilde{x}}(\tilde{y}, w - \Pi_{\tilde{x}+\tilde{y}}(\tilde{z}, w)) + \tilde{x}), \end{aligned}$$

from which it follows that

$$\Pi_{\tilde{x}}(\tilde{y} + \tilde{z}, w) = \Pi_{\tilde{x}+\tilde{y}}(\tilde{z}, w) + \Pi_{\tilde{x}}(\tilde{y}, w - \Pi_{\tilde{x}+\tilde{y}}(\tilde{z}, w)). \tag{9}$$

In particular, when $\tilde{x} = 0$, the above equation writes as

$$\pi(\tilde{y} + \tilde{z}, w) = \Pi_{\tilde{y}}(\tilde{z}, w) + \pi(\tilde{y}, w - \Pi_{\tilde{y}}(\tilde{z}, w)). \tag{10}$$

That is, the costs of multiple risks can be decomposed into the cost of the first risk evaluated in the presence of the second risk and the cost of the second risk evaluated with a sure reduction to wealth due to the existence of the first risk.

In order to characterize the equilibrium sorting pattern, by (6), we need to compare $\pi(0, w_0) + \pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0)$ and $\pi(\tilde{\varepsilon}^m, w_0) + \pi(\tilde{\varepsilon}^f, w_0)$.In their seminal paper, Gollier and Pratt (1996) have introduced the concept of risk vulnerability as a basic tool to examine the effect of unfair background risk on agent's attitude towards other independent risk. In particular, *utility is **risk vulnerable*** iff $\Pi_{\tilde{\varepsilon}}(\tilde{x}, w) \geq \pi(\tilde{x}, w)$ for *all w and unfair $\tilde{\varepsilon}$ ($E\tilde{\varepsilon} \leq 0$).That is, the introduction of an unfair $\tilde{\varepsilon}$ ($E\tilde{\varepsilon} \leq 0$) increases* *the risk premium of every independent risk $\tilde{x}$.* Gollier and Pratt (1996) have listed several necessary and sufficient conditions for risk vulnerability, among which, under HARA, a sufficient condition for risk vulnerability is DARA.[1] With the above tool, we have the sorting result for this degenerated case in the following proposition.

Proposition 2a *If preference belongs to the HARA class and exhibits DARA, and agents are endowed with $w_1^m = w_1^f = w_0$, $\tilde{w}_2^m = w_0 + \tilde{\varepsilon}^m$, $\tilde{w}_2^f = w_0 + \tilde{\varepsilon}^f$ where $E\tilde{\varepsilon}^m = E\tilde{\varepsilon}^f = 0$, the 2×2 risk-sharing match game will be negative assortative on spouse's income riskiness.*

Proof. First notice that $\pi(0, w_0) = 0$. We want to prove $\pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) \geq \pi(\tilde{\varepsilon}^m, w_0) + \pi(\tilde{\varepsilon}^f, w_0)$. Under ISHARA and DARA, utility is risk vulnerable and we have

$$\Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w_0) > \pi(\tilde{\varepsilon}^f, w_0) \tag{11}$$

By DARA, we have

$$\pi(\tilde{\varepsilon}^m, w_0 - \Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w_0)) > \pi(\tilde{\varepsilon}^m, w_0) \tag{12}$$

By (10)

$$\pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) = \Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w_0) + \pi(\tilde{\varepsilon}^m, w_0 - \Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w_0))$$

Combining with (11) and (12), we have $\pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) > \pi(\tilde{\varepsilon}^f, w_0) + \pi(\tilde{\varepsilon}^m, w_0)$ and thus conclude that stable match is negative assortative on income riskiness. ■

Under HARA utility, DARA is necessary and sufficient for risk vulnerability. Consider the couple both with risky income. Firstly, as in (11), risk vulnerability guarantees that the introduction of a pure risk $\tilde{\varepsilon}^m$ increases the cost of existing independent risk $\tilde{\varepsilon}^f$. Secondly, as in (12), DARA implies that the presence of this pure risk $\tilde{\varepsilon}^m$ increases the cost of the pure risk itself. In this sense, multiple risks are self aggravating under DARA. Hence negative sorting is socially preferable and stable.

It is important to mention that, while risk vulnerability states that $\Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w) \geq \pi(\tilde{\varepsilon}^f, w)$, the condition for NAM requires $\pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) - \pi(\tilde{\varepsilon}^m, w_0) \geq \pi(\tilde{\varepsilon}^f, w_0)$. Hence, we still need to prove that $\pi(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) - \pi(\tilde{\varepsilon}^m, w_0) \geq \Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w) \iff \pi(\tilde{\varepsilon}^m, w_0 - \Pi_{\tilde{\varepsilon}^m}(\tilde{\varepsilon}^f, w)) \geq \pi(\tilde{\varepsilon}^m, w_0)$,which makes DARA a sufficient condition. In the more general case, risk vulnerability might be not sufficient to guarantee NAM.

[1]See Gollier and Pratt (1996) COROLLARY 1, page 117.

5.3 Rothschild-Stiglitz Case with Independent Noise

Now we consider the 2×2 case where $\tilde{\varepsilon}_2^m$ is an increase in risk of $\tilde{\varepsilon}_1^m$ in the sense of Rothschild-Stiglitz taking the form of adding an independent noise $\tilde{\varepsilon}^m$, and similarly for $\tilde{\varepsilon}_2^f$ and $\tilde{\varepsilon}_1^f$. That is, assume m_1 and f_1 to be endowed with $\tilde{w}_1^m = w_0 + \tilde{\varepsilon}_1^m$ and $\tilde{w}_1^f = w_0 + \tilde{\varepsilon}_1^f; m_2$ and f_2 to be endowed with $\tilde{w}_2^m = w_0 + \tilde{\varepsilon}_1^m + \tilde{\varepsilon}^m$ and $\tilde{w}_2^f = w_0 + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^f$. All idiosyncratic risks and noises are independently distributed and $E\tilde{\varepsilon}_1^m = E\tilde{\varepsilon}^m = E\tilde{\varepsilon}_1^f = E\tilde{\varepsilon}^f = 0$. Here, in order to characterize the equilibrium sorting pattern, by (6), we need to compare $\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0)$ and $\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^m, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^f, w_0)$. Before proceeding, the following Lemma is useful.

Lemma 3 *If preference belongs to the HARA class, household risk premium $\pi(\tilde{\varepsilon}_i^m + \tilde{\varepsilon}_j^f, w_0)$ is convex in w_0.*

Proof. Similar to the proof of proposition 1. ∎

Proposition 2b *If preference belongs to the HARA class and exhibits DARA, the risk-sharing matching game will be negative assortative on spouse's income riskiness in the sense of Rothschild-Stiglitz taking the form of adding an independent noise.*

Proof. Stable match is negative assortative if

$$\begin{aligned} & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) \\ \geq \;& \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^m, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^f, w_0) \end{aligned}$$

which under (10) is equivalent to

$$\begin{aligned} & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) \\ + \;& \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0)) \\ \geq \;& \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0)) \\ + \;& \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0)) \end{aligned} \tag{13}$$

By applying (9), we have

$$\begin{aligned} & \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) \\ = \;& \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^f}(\tilde{\varepsilon}^m, w_0) + \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f + \tilde{\varepsilon}^f}(\tilde{\varepsilon}^m, w_0)) \end{aligned} \tag{14}$$

Under risk vulnerability we have

$$\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f+\tilde{\varepsilon}^f}(\tilde{\varepsilon}^m, w_0) \geq \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0) \tag{15}$$

And since DARA is preserved under generalized risk premium, we have

$$\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f+\tilde{\varepsilon}^f}(\tilde{\varepsilon}^m, w_0)) \geq \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0) \tag{16}$$

Combining (14), (15), (16), we have

$$\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0) \geq \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0) + \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0). \tag{17}$$

Under DARA, $\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$ is decreasing in w_0, which gives

$$\begin{aligned} & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0)) \\ \geq \; & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0) - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0)) \end{aligned} \tag{18}$$

Combining (13) and (17), stable match satisfied NAM if

$$\begin{aligned} & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m + \tilde{\varepsilon}^f, w_0)) \\ \geq \; & \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^m, w_0)) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon}^f, w_0)) \end{aligned}$$

Under (17) and (18), a sufficient condition is $\pi(\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f, w_0)$ being convex in w_0, which is indeed the case by Lemma 3. ■

Again, under HARA utility, DARA is necessary and sufficient for risk vulnerability. Risk vulnerability guarantees that the Rothschild-Stiglitz deterioration taking the form of adding independent noise increases the cost of existing independent risks. DARA implies that the Rothschild-Stiglitz deterioration taking the form of adding independent noise increases the cost of the deterioration itself. Multiple risks are in this sense self aggravating and thus negative sorting is socially preferable and stable.

5.4 General Rothschild-Stiglitz Case

In this section, we examine the general case where $\tilde{\varepsilon}_2^m \precsim \tilde{\varepsilon}_1^m$ and $\tilde{\varepsilon}_2^f \precsim \tilde{\varepsilon}_1^f$ in the Rothschild-Stiglitz sense. Remind that by Lemma 2 the stable match is NAM if the following supermodular condition holds:

$$\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m, w_0) \geq \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f, w_0). \tag{19}$$

Because $\tilde{\varepsilon}_2^m \precsim \tilde{\varepsilon}_1^m$, $\tilde{\varepsilon}_2^f \precsim \tilde{\varepsilon}_1^f$ and $\tilde{\varepsilon}_i^m$s, $\tilde{\varepsilon}_i^f$s are independent, we have the order: $\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m \precsim \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \precsim \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f$, $\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m \precsim \tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \precsim \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f$. Moreover, it's obvious that $\left[\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f\right] + \left[\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m\right] = \left[\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f\right] + \left[\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f\right]$. The supermodular condition (19) is similar, though not equivalent, as to say that the risk premium is "convex" in riskiness of idiosyncratic risks.

Consider the case where $\tilde{\varepsilon}_2^f \succsim \tilde{\varepsilon}_1^f$, and $\tilde{\varepsilon}_2^m$ is an increase in risk of $\tilde{\varepsilon}_1^m$ in the sense of Rothschild-Stiglitz taking the form of adding an independent nose $\tilde{\varepsilon}$, i.e., $\tilde{\varepsilon}_2^m = \tilde{\varepsilon}_1^m + \tilde{\varepsilon}$. That is, we only impose independent noise assumption on one side of the population. Stable match satisfies NAM if

$$\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon} + \tilde{\varepsilon}_1^f, w_0) - \pi(\tilde{\varepsilon}_1^m + \varepsilon_1^f, w_0) \leq \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon} + \tilde{\varepsilon}_2^f, w_0) - \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0)$$

which is equivalent to

$$\begin{aligned} & \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}, w_0) + \pi(\tilde{\varepsilon}_1^m + \varepsilon_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}, w_0)) - \pi(\tilde{\varepsilon}_1^m + \varepsilon_1^f, w_0) \\ \leq\ & \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f}(\tilde{\varepsilon}, w_0) + \pi(\tilde{\varepsilon}_1^m + \varepsilon_2^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f}(\tilde{\varepsilon}, w_0)) - \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0). \end{aligned}$$

Now we prove

$$\Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}, w_0) \leq \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f}(\tilde{\varepsilon}, w_0), \tag{20}$$

and

$$\begin{aligned} & \pi(\tilde{\varepsilon}_1^m + \varepsilon_1^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f}(\tilde{\varepsilon}, w_0)) - \pi(\tilde{\varepsilon}_1^m + \varepsilon_1^f, w_0) \\ \leq\ & \pi(\tilde{\varepsilon}_1^m + \varepsilon_2^f, w_0 - \Pi_{\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f}(\tilde{\varepsilon}, w_0)) - \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0). \end{aligned} \tag{21}$$

Eeckhoudt, Gollier and Schlesinger (1996) provide a full characterization of the comparative results with respect to change in background risk. In particular, they prove that the "Ross risk vulnerability" conditions are necessary and sufficient for any SSD deterioration in the background to raise local risk aversion at w. The "Ross risk vulnerability" conditions are given as follows:

1) u exhibits decreasing absolute risk aversion in the sense of Ross over the relevant range of wealth, i.e., there exists a scalar λ, such that

$$p\left(w+y\right) \geq \lambda \geq r\left(w+y'\right), \forall y, y', \tag{22}$$

where $p\left(w\right) = \frac{-u'''(w)}{u''(w)}$ denotes the measure of absolute prudence and $r\left(w\right) = \frac{-u''(w)}{u'(w)}$ denotes the measure of absolute risk aversion.

2) there exists a scalar η such that

$$t\left(w+y\right)\geq\eta\geq r\left(w+y'\right),\ \forall y,y', \tag{23}$$

where $t\left(w\right)=\frac{-u''''(w)}{u'''(w)}$ denotes the measure of absolute temperance.

If the above "Ross risk vulnerability" conditions hold, any SSD deterioration in the background makes the agent more risk averse and hence (20) holds. Under decreasing absolute prudence, $t(w)$ is larger than $p(w)$ for all w and hence condition 1) implies condition 2). Moreover, if utility belongs to the HARA class and exhibits DARA in the sense of Ross, then the absolute prudence is indeed decreasing. Therefore, if the utility function belongs to HARA and exhibits DARA in the sense of Ross, then (20) holds.

The utility function belonging to HARA and exhibiting DARA in the sense of Ross combined is also a sufficient condition for inequality (21) to hold, as:

$$\begin{aligned}
& \pi(\tilde{\varepsilon}_1^m+\varepsilon_2^f,w_0-\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_2^f}(\tilde{\varepsilon},w_0))-\pi(\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_2^f,w_0)\\
\geq\ & \pi(\tilde{\varepsilon}_1^m+\varepsilon_2^f,w_0-\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon},w_0))-\pi(\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_2^f,w_0)\\
\geq\ & \pi(\tilde{\varepsilon}_1^m+\varepsilon_1^f,w_0-\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon},w_0))-\pi(\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f,w_0).
\end{aligned}$$

The first inequality comes from the fact that π is decreasing in w_0 and that $\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_2^f}(\tilde{\varepsilon},w_0)>\Pi_{\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f}(\tilde{\varepsilon},w_0)$.The second inequality is a direct result of the fact that $\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_2^f\preceq\tilde{\varepsilon}_1^m+\tilde{\varepsilon}_1^f$ and of the following Lemma.

Lemma 4 *If utility exhibits DARA in the sense of Ross, $\tilde{\varepsilon}_2$ and $\tilde{\varepsilon}_1$ are risks such that $\tilde{\varepsilon}_1\succeq\tilde{\varepsilon}_2$ in the Rothschild-Stiglitz sense, then $\pi(\tilde{\varepsilon}_2,w_0)-\pi(\tilde{\varepsilon}_1,w_0)$ is decreasing in w_0.*

Proof. As in Gollier (2001), we define the risk premium $\pi(\tilde{\varepsilon}_2\to\tilde{\varepsilon}_1,w_0)$ as the price that agent is ready to pay to replace $\tilde{\varepsilon}_2$ by $\tilde{\varepsilon}_1$ when his initial wealth is w_0, i.e.,

$$Eu\left(w_0+\tilde{\varepsilon}_2\right)=Eu\left(w_0-\pi(\tilde{\varepsilon}_2\to\tilde{\varepsilon}_1,w_0)+\tilde{\varepsilon}_1\right),$$

from which we have

$$\pi\left(\tilde{\varepsilon}_2,w_0\right)=\pi(\tilde{\varepsilon}_2\to\tilde{\varepsilon}_1,w_0)+\pi\left(\tilde{\varepsilon}_1^m,w_0-\pi(\tilde{\varepsilon}_2\to\tilde{\varepsilon}_1,w_0)\right). \tag{24}$$

The above equation suggests that the cost of a large risk can be paid to replace by two steps. The first step is to pay a certain amount of money to replace the risk by a small risk. The second step is to pay a certain amount of money to remove this small risk. Hence, the cost of this small risk should be evaluated at a sure reduction of wealth due to the first step.

We need to prove $\pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0) + \pi\left(\tilde{\varepsilon}_1, w_0 - \pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right) - \pi(\tilde{\varepsilon}_1, w_0)$ is decreasing in w_0. Since utility exhibits DARA in the sense of Ross. From Gollier (2001, P109: Proposition 22), we know that $\pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)$ is decreasing in w_0. Moreover,

$$
\begin{aligned}
& \frac{\partial}{\partial w_0}\{\pi\left(\tilde{\varepsilon}_1, w_0 - \pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right) - \pi(\tilde{\varepsilon}_1, w_0)\} \\
= & \frac{\partial}{\partial w_0}\pi\left(\tilde{\varepsilon}_1, w_0 - \pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right) - \frac{\partial}{\partial w_0}\pi(\tilde{\varepsilon}_1, w_0) \\
& -\left(\frac{\partial}{\partial w_0}\pi\left(\tilde{\varepsilon}_1, w_0 - \pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right)\frac{\partial}{\partial w_0}\pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right) \\
= & -\left[\begin{array}{c} \int_{w_0-\pi(\tilde{\varepsilon}_2\to\tilde{\varepsilon}_1, w_0}^{w_0} \frac{\partial^2}{\partial w_0^2}\pi(\tilde{\varepsilon}_1, w)dw \\ +\frac{\partial}{\partial w_0}\pi\left(\tilde{\varepsilon}_1, w_0 - \pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)\right)\frac{\partial}{\partial w_0}\pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0) \end{array}\right] < 0.
\end{aligned}
$$

The last inequality holds due to the facts that 1) $\pi(\tilde{\varepsilon}_1, w_0)$ is convex in w_0; 2) $\pi(\tilde{\varepsilon}_1, w_0)$ is decreasing in w_0; 3) $\pi(\tilde{\varepsilon}_2 \to \tilde{\varepsilon}_1, w_0)$ is decreasing in w_0. ■

An equivalent way to say that utility exhibits DARA in the sense of Ross is that there exists a positive scalar λ and a decreasing and concave function g such that $v(w_1 + z) = \lambda v(w_2 + z) + g(z)$ for all z and for all $w_1 < w_2$ (Ross, 1981).

Under our HARA assumption on preference, utility function can be written as $u(c) = \left(c + \frac{\gamma}{\alpha}\right)^{1-\gamma}$. Then, (22) becomes

$$\frac{\gamma+1}{(w+y)+\frac{\gamma}{\alpha}} \geq \lambda \geq \frac{\gamma}{(w+y')+\frac{\gamma}{\alpha}}, \ \forall y, y'.$$

Suppose the relevant range of wealth is bounded on the interval $[a, b]$. Then, the above inequality becomes

$$\frac{\gamma+1}{b+\frac{\gamma}{\alpha}} \geq \frac{\gamma}{a+\frac{\gamma}{\alpha}},$$

which can be simplified to

$$b - a \leq \frac{1}{\alpha} + \frac{1}{\gamma}a. \tag{25}$$

When the range of the relevant wealth is not too large, the utility exhibits DARA in the sense of Ross. Consequently, NAM arises if we impose independent noise assumption on one side of the population.

More generally, when no independent noise assumption is imposed, we have the following sufficient condition for NAM to arise.

Proposition 2c *If preference belongs to the HARA class, and any Rothschild-Stiglitz deterioration in background risk raises risk aversion in the sense of Ross, then the risk-sharing matching game will be negative assortative on spouse's income riskiness in the sense of Rothschild-Stiglitz.*

Proof. Stable match is NAM if

$$\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_2^f, w_0) \geq \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f, w_0). \tag{26}$$

By applying (24), the above inequality can be rewritten as

$$\begin{aligned} & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_1)\right] \\ \geq \; & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_2)\right], \end{aligned} \tag{27}$$

where $w_1 = w_0 - \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0)$ and $w_2 = w_0 - \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$.

Now we prove

$$\pi(\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) \geq \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0). \tag{28}$$

Consider agent 1 with utility function $V_1(x) = Ev(x + \tilde{\varepsilon}_1^f)$ and agent 2 with $V_2(x) = Ev(x + \tilde{\varepsilon}_2^f)$. Like Gollier (2001), we define the risk premium $\pi_1(\tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m)$ as the price that agent 1 is ready to pay to replace $\tilde{\varepsilon}_2^m$ by $\tilde{\varepsilon}_1^m$, and $\pi_2(\tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m)$ as the counterpart for agent 2. Then $\pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) = \pi_1(\tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m)$ and $\pi(\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) = \pi_2(\tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m)$. (28) holds iff V_2 is more risk averse than V_1 in the sense of Ross, i.e., a Rothschild-Stiglitz deterioration in the background risk makes the agents more risk averse in the sense of Ross.

Given (28), (27) holds if

$$\begin{aligned} & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_1)\right] \\ \geq \; & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_2)\right] \end{aligned} \tag{29}$$

which, by applying (24), can be rewritten as

$$\begin{aligned} & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_4) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_1)\right] \\ \geq \; & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_3) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_2)\right] \end{aligned}$$

where $w_4 = w_1 - \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_1)$, $w_3 = w_0 - \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$. If utility exhibits DARA in the sense of Ross, we have $\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_1) \geq \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$. Thus, we only need to prove

$$\begin{aligned} & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_4)\right] \\ \geq \; & \left[\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_2) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_3)\right]. \end{aligned} \tag{30}$$

To prove (30), we first prove

$$w_0 + w_4 \leq w_2 + w_3. \tag{31}$$

Using the expression of w_0, w_2, w_3 and w_4, (31) can be rewritten as

$$\begin{aligned} & \left[\pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_1)\right] \\ \geq \; & \left[\pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0) + \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)\right]. \end{aligned}$$

By (**??**), we know $\pi(\tilde{\varepsilon}_2^f + \tilde{\varepsilon}_2^m \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f, w_0) \geq \pi(\tilde{\varepsilon}_2^m + \tilde{\varepsilon}_1^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$. Moreover, that the utility is DARA in the sense of Ross and $w_0 > w_1$ implies $\pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_1) \geq \pi(\tilde{\varepsilon}_1^m + \tilde{\varepsilon}_2^f \to \tilde{\varepsilon}_1^m + \tilde{\varepsilon}_1^f, w_0)$. Hence, the above inequality and therefore (31) holds.

Inequality (30) follows by noticing the facts that (1) π is decreasing and convex in w_0; (2) $w_0 > \max(w_2, w_3) > \min(w_2, w_3) > w_4$; (3) inequality (31) holds. ■

Remark What remains to be done is to find restrictions on utility so that any Rothschild-Stiglitz deterioration raises local risk aversion in the sense of Ross. V_2 is more risk averse than V_1 in the sense of Ross iff there exists a scalar η such that

$$\frac{Ev''(w_1 + \tilde{\varepsilon}_2)}{Ev''(w_1 + \tilde{\varepsilon}_1)} \geq \eta \geq \frac{Ev'(w_2 + \tilde{\varepsilon}_2)}{Ev'(w_2 + \tilde{\varepsilon}_1)}, \forall w_1, w_2. \tag{32}$$

For small risk $\tilde{\varepsilon}_i$, we have

$$\begin{array}{l} Ev''(w + \tilde{\varepsilon}_i) \approx v''(w) + \frac{1}{2} v''''(w)\,\sigma_i^2 \\ Ev'(w + \tilde{\varepsilon}) \approx v'(w) + \frac{1}{2} v'''(w)\,\sigma_i^2 \end{array}.$$

Substitute the above expression into the condition (32), we have

$$\frac{v''''(w_1)}{v''(w_1)} \geq \eta \geq \frac{v'''(w_2)}{v'(w_2)}, \ \forall\forall w_1, w_2.$$

Or equivalently,

$$t(w_1)\, p(w_1) \geq \eta \geq p(w_2)\, r(w_2), \ \forall w_1, w_2, \tag{33}$$

where $t(w) = \frac{-u''''(w)}{u'''(w)}$ denotes the measure of absolute temperance, $p(w) = \frac{-u'''(w)}{u''(w)}$ denotes the measure of absolute prudence, and $r(w) = \frac{-u''(w)}{u'(w)}$ denotes the measure of absolute risk aversion. Under our specification on utility function, (33) becomes

$$\frac{\gamma + 2}{\left(w_1 + \frac{\gamma}{\alpha}\right)^2} \geq \frac{\gamma}{\left(w_2 + \frac{\gamma}{\alpha}\right)^2}, \forall w_1, w_2,$$

which will hold if the support of w_1, w_2 is sufficiently narrow. In general, for large risks, deriving conditions for equation (32) is complex and we leave it for future work.

6 Concluding Remarks

When market situations are characterized by two sides of population who are heterogeneous in their income riskiness and try to match with appropriate partners for consumption insurances, this paper shows that competitive equilibrium sorting is characterized by NAM under considerably realistic hypotheses on risk preference. Our findings enrich the literature of endogenous group formation for the purpose of risk sharing, and to the best of our knowledge, are among the first attempts to assume differentiation on risks.

The present research can be extended along several lines. In the actual economy, household decision is made dynamically which permit individuals to choose endogenous risk posterior to the matching stage. There could then be a trade-off between competing for most suitable partner for the purpose of risk sharing and for the motive of risk control.

The present research is carried out under linear risk tolerance. With this restriction, monetary equivalent can be losslessly transferred from one spouse to the other. Thus the interaction of risks in the household's portfolio only depends on the differential effect of deterioration of risk to the household risk premium. Other than the linear risk tolerance, the efficient risk sharing rules which map household income into individual consumption are typically nonlinear. It is, however, worthwhile to find less sufficient restrictions on preference for the current predictions to hold. In a nontransferable utility framework with no pre-existing hypothesis on preference, Sun and Li (2010) provide some necessary or sufficient conditions which require the supports of all risks to be not too large with respect to initial asset position and/or risk tolerance to be sufficiently linear.

Understanding the effect of risk factors on household composition is also important for appropriate design of economic policies. Households are recognizably constituted of actors with conflicting interests and thus, in practice, the ultimate intrahousehold allocation often depends on the spouses' relative bargaining powers and their differentiated threat points. Legal restrictions and social norms may affect household opportunities to cope with risk by changing the spouses' exit options from informal insurance. Law economists have long suspected that divorce laws help shape marriage institutions. In the 1970s and early 1980s, the United States underwent a liberal shift of divorce laws which eased the unilateral resolution. Some might argue that the compensation principle implicit in the rule of efficient divorce suggests that there are no efficiency issues in the laws. However, to the extent that legal rules affect the joint cost of divorce and the residual fall-back positions even when the marriage does not actually dissolve. Divorce laws are relevant as they might affect people's ex ante decision on the marriage market. The welfare evaluation of such legislative reforms cannot be studied without explicitly grounding the composition of risk-sharing partnerships in a structural model.

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