# The yield curve as a leading indicator in economic forecasting in the UK

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**Abstract**

This paper investigates a model utilising the term structure of interest rates to predict output growth and recession in the UK. In contrast to previous literature, information retrieved from the whole yield curve is used rather than just the yield spread. Using different methods, our models are found to outperform the yield spread models both in in-sample and out-of-sample forecasting. Notably, the B-spline fitting model is able to forecast the 2009-2010 recession. Moreover, Model B show great forecasting ability in out-of-sample output growth forecasting. In most cases, models based on B-spline perform better than the ones based on Diebold-Li framework.

*JEL Classification:* E27; E37

*Keywords:* Term structure, Forecast, Real growth, Recession, UK

## 1. Introduction

The practical objective for economic forecasting is to provide policymakers with new economic tools to estimate the impact on aggregate activities of their potential decisions. The importance of accurate forecasting was brought into sharp focus long ago by the painful experience of the Great Depression. However, this is not saying that a good forecaster can prevent a recession like the recent financial crisis happening, but a good forecaster may help to reduce the loss caused by recession to an acceptable level. Nevertheless, the regulator is not the only party who can benefit from forecasting but

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also private agents such as practitioners, portfolio managers and risk managers whose future earnings and business strategies will be influenced by the quality of such forecasts.

The term structure of interest rates has been mentioned frequently in the context of monetary policy, particularly as an indicator of market expectations or of the position of policy. Although it is rarely viewed as a policy target, it is generally conceded to contain some information that may be of use to both market participants and to the monetary authority. There has already been a relatively extensive literature examination of the informational and predictive content of the term structure with regard to the conventional final targets of monetary policy, which are inflation and real activity. For instance, when people expect a recession, they will change their investment behaviour: withdraw money from short-term investing and put them into long-term investing. In bond market this behaviour leads to a higher long-term bond price and a lower short-term bond price, so the yield of short-term bonds will rise and the yield on long-term bonds will drop, changing the shape of the yield curve.

This paper looks at economic forecasting through the term structure of interest rates from a new perspective by examining the whole yield curve and using the information to forecast recessions and output growth.

In the literature, there has been little research using the whole yield curve. The majority use only the slope or both slope and level to investigate the relationship between yield curve and output growth. Our contribution is threefold: First, conventionally, researchers use yield spread to represent the yield curve. However, the yield spread is essentially based on the assumption that the yield curve is a straight line, while it may be that the non-linearity in the yield curve embodies predictive power. Therefore, the innovative feature of our paper is that this assumption is relaxed by using the whole yield curve. Second, we use two different approaches, parametric and non-parametric to model the yield curve in order to meet a great variety of forecasting purposes. Third, we demonstrate that the term structure forecasting model has excellent forecast ability in recession forecasting. This is particularly relevant in the sight of the major recession recently experienced. It is especially crucial since we have experienced a big recession recently.

The real-time zero-coupon rate and real GDP growth in the UK for the period from 1979q4 to 2009q4 are used in this paper. We adopt the definition of recession used by the National Bureau of Economic Research (NBER) which is a period in which GDP falls (negative real economic growth) for at least two consecutive quarters. There are two striking features in the results of this study: 1. The B-spline and Diebold-Li frameworks forecast fit better than the yield spread model and achieve higher forecast performance in short-horizon forecasting especially 2-quarter ahead forecast. In contrast to term spread forecasting, both Diebold-Li framework or B-spline framework based on the whole yield curve show stable performance from 1 to 12-quarter ahead forecasting. Models based on the whole yield curve outperform those based on yield spread both in-sample and out-of-sample test in output forecasting. 2. In terms of forecasting ability of different forecasting approaches under two frameworks (Diebold-Li framework and B-spline framework), the out-of-sample forecasting results demonstrated that probit model base on B-spline approach exhibits extremely high forecast ability for forecasting recessions.

The rest of this paper is structured as follows: Section 2 is a brief literature review about using yield curve to forecast economic growth. In Section 3 we illustrate the methods we use to construct a yield curve and the forecasting models. Section 4 is the description of the data we choose. The results and findings of our research is presented in Section 5. Section 6 concludes.

## 2. Brief Literature Review

The use of interest rates and their term structure as a predictor of recession and GDP growth has been widely studied, and these literature show strong evidence that it is reliable (Fama, 1990; Mishkin, 1990; Estrella & Hardouvelis, 1991; Zagaglia, 2006; Bordo & Haubrich, 2008). However, evidence shows that not all the countries in the world can use term structure of interest rate as a leading indicator, while most of the literature find that the UK is one of those that can use it (Jorion & Mishkin, 1991; Harvey, 1991; Estrella & Mishkin, 1997; Plosser & Rouwenhorst, 1994; Bernard & Gerlach, 1998). Mishkin (1990) concludes that term spread is not significant in a big part of OECD countries, except the UK, France and Germany. Schich (2000) finds in G-7 countries, US, Germany, UK and Canada yield spreads are significant for output forecasting. Bonser & Morley (1997) show evidence that UK Canada and Germany can use yield spread as a leading indicator, but weak evidence is shown in Japan and Switzerland.

The information included in the term structure successfully predicts recessions with discrete choice models, in which the recession is coded as 1 and other times coded 0 (Estrella & Mishkin, 1999; Wright, 2006). Although Dotsey (1998), and Stock & Watson (2003) report that the predictive power of the spread has decreased after 1985, Estrella & Mishkin (1999)'s work demonstrates that spread is still better than other leading indicators in predicting recession. In Stock & Watson (1999)'s work they include term spread as a very important element in their leading business cycle indicator index. By introducing monetary regime into the explanatory variables, Bordo & Haubrich (2004) successfully increase the predictive ability of yield curve and show that this influence is changing over time (Stock & Watson, 2003), then they imply a short-term interest rate (short-term commercial paper rate) to improve the predictive power of the whole model. Ang et al. (2006) test the spreads between different long-term bonds to a 3-month bond together using a VAR approach. This approach avoids the limitation of using a 10-year and a 3-month spread and predicts that greater explanatory power should come from longer term spreads.

In order to add more elements into forecasting models, we apply a Nelson & Siegel (1987) exponential components framework modified by Diebold & Li (2006) and the B-spline model (de Boor, 1978) to include more information in the yield curve in the UK economy. Diebold-Li framework and B-spline model can properly model yield from 3-month 6-month 9-month... to 15 year all in one daily yield curve. Moreover, from the Diebold-Li framework, this curve has three estimators which represent short-, medium- and long-term yield. The B-spline model is a non-parametric model which fits a curve very well. In our research, we use this whole yield curve to forecast both the real growth and the recession to identify the forecast ability of the whole yield curve.

## 3. Methodology

The econometric modelling approach adapted here consist of two stages: The first stage is to model the yield curve and the second stage is using the estimators obtained from the first stage to forecast recession and real output growth.

### *3.1. Stage 1: Model the yield curve*

Several approaches have been developed for modeling the yield curve. The Bank of England for example have adapted a model developed by Mastronikola (1991) to estimate term structure in the early 1990s, later replacing it with a parametric model developed by Nelson & Siegel (1987), and then further improved by Svensson (1994). However, Fisher et al. (1995) and Waggoner (1997) construct term structure using non-parametric models based on cubic splines (B-splines) which later became the official model used by the Bank of England. Anderson & Sleath (2008) compared these models and conclude that the Nelson & Siegel (1987) model appears to be much more stable than the Svensson technique, while in all cases the Waggoner (1997) curve appears to perform well. Practically, the Nelson and Siegel model modified by Diebold & Li (2006) appears to be more stable than the Svensson model and more flexible than the Nelson-Siegel approach. Therefore, we choose one parametric model which is Diebold-Li framework for explanatory purpose and one non-parametric model which uses the B-spline technique to construct yield curve for forecasting purposes.

*3.1.1. Diebold-Li framework*

Diebold-Li framework is improving the Nelson-Siegle model by solving the two main pitfalls which are hard to disassemble and explain the effect or significance of the two factors and difficult to estimate the factors precisely because the high coherence in the factors produces multicollinearity. The framework is as follows:

$$y_t(\tau) = \beta_{1t} + \beta_{2t}\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} + \beta_{3t}\left(\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}\right) \tag{1}$$

Where $y_t(\tau)$ is yield at time $t$ of a bond with time to maturity $\tau$.

Because the correlation between $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau}$ and $\frac{1-e^{-\lambda_t\tau}}{\lambda_t\tau} - e^{-\lambda_t\tau}$ is greatly reduced, the multicollinearity has been solved. The four parameters $\beta_{1t}$, $\beta_{2t}$, $\beta_{3t}$ and $\lambda_t$ can be interpreted as long-, short-, medium-term factors respectively and exponential decay rate which represent the level, slope, curvature and proportion between slope and curvature of the curve respectively.

Figure 1 shows how the model decomposes the yield curve into 3 factors.

For simplicity, Diebold & Li (2006) fix $\lambda_t$ at 0.0609. At this value the medium-term, or curvature, factor achieves its maximum at a maturity of 30 months. This considered as standard.

*3.1.2. B-spline model*

A generic spline is a piecewise polynomial, i.e. a curve constructed from individual polynomial segments joined at 'knot points', with coefficients chosen such that the curve and its first derivative are continuous at all points. The most commonly used polynomials are cubic functions – giving a cubic spline. The continuity constraints imply that any cubic spline can be written in the form:

$$S(x) = \alpha x^3 + \beta x^2 + \gamma x + \delta + \sum_{i=1}^{N-1} \eta_i \left| x - k_i \right|^3 \tag{2}$$

for some constants $\alpha$, $\beta$, $\gamma$, $\delta$, $\eta_i$,where $k_i$ is the $i$th knot and $N$ is the number of knots are chosen. It is the simplest expression for a cubic spline, but numerically unstable, and therefore a linear combination of cubic B-spline is preferred instead. This is a completely general transformation (any spline can be written as such a combination of B-splines of the appropriate order), which solves the numerically unstable problem. B-splines of order $n$ are most simply represented by the following recurrence relation:

$$B_{i,n}(x) = \frac{x - k_i}{k_{i+n-1} - k_i} B_{i,n-1}(x) + \frac{k_{i+n} - x}{k_{i+n} - k_{i+1}} B_{i+1,n-1}(x) \tag{3}$$

with $B_{i,1}(x) = 1$ if $k_i \leq x < k_{i+1}$, and $B_{i,1}(x) = 0$ otherwise.[2]

In our case 1 internal knot[3], $k_1 = 90$ is used and the yield curve can be written as:

$$y_t(\tau) = bs_{1t}B_1(\tau) + bs_{2t}B_2(\tau) + bs_{3t}B_3(\tau) + bs_{4t}B_4(\tau) + bs_{5t}B_5(\tau) \tag{4}$$

where $B_1(\tau)$, $B_2(\tau)$, $B_3(\tau)$, $B_4(\tau)$ and $B_5(\tau)$ are b-splines according to the internal knot. Figure 2 shows the decomposition of yield curve into B-splines.

*3.2. Stage 2: Forecasting Model*

By estimating $\beta_{1t}$, $\beta_{2t}$ and $\beta_{3t}$ from equation (1) and $bs_{1t}$, $bs_{2t}$, $bs_{3t}$, $bs_{4t}$ and $bs_{5t}$ from equation (4), information is extracted from the yield curve on the last day of the each quarter. We adapted the following forecasting models to draw a connection with recession variable or real GDP growth and yield curve.

[2]For further details see Lancaster & Salkauskas (1986)

[3]Here using 1 internal knot in B-spline model, for balancing the accuracy of the curve modelling and the degree of freedom in the forecasting models.

*3.2.1. Recession forecasting*

We suggest a probit model involves the prediction of whether or not the economy will be experiencing a recession $k$ quarters ahead. This model abstracts from the actual magnitude of economic activity by focusing on the simple binary indicator variable. Although this forecast is in some sense less precise, the requirements on predictive power are in another sense less demanding and may increase the potential accuracy of the more limited forecast. Here the NBER definition of recession is used. The value of probability of recession equal to 1 when the economy is in recession, and 0 when it is not in recession. And the models are as follow:

For Diebold-Li framework:

$$P(\text{recession}_t) = \Phi(\alpha_1\beta_{1,t-h} + \alpha_2\beta_{2,t-h} + \alpha_3\beta_{3,t-h}) \tag{5}$$

For B-spline model:

$$P(\text{recession}_t) = \Phi(\alpha_1 bs_{1,t-h} + \alpha_2 bs_{2,t-h} + \alpha_3 bs_{3,t-h} + \alpha_4 bs_{4,t-h} + \alpha_5 bs_{5,t-h}) \tag{6}$$

where $h$ is the forecasting horizon. The value of probability of recession equal to 1 when the economy is in recession, and 0 when it is not in recession.

*3.2.2. Real GDP growth forecasting*

All the annual GDP growth calculated as follows:

$$\Delta y_t = \log(y_t) - \log(y_{t-4}) \tag{7}$$

We apply two different approaches to forecast the real output growth. They are identified as Model A and B. The Model A is a model with the real GDP growth being dependent variable and lagged yield curve representative variables being independent variables. And the equation is as follows:

$$\text{Model A:} \quad \Delta y_t = AX_{t-h} + \epsilon_t \tag{8}$$

where matrix $X = (\beta_1, \beta_2, \beta_3)$ for Diebold-Li framework and matrix $X = (bs_1, bs_2, bs_3, bs_4, bs_5)$ for B-spline model.

Model B is set up as follows:

$$\text{Model B:} \quad \Delta y_t = \phi \Delta y_{t-h} + AX_{t-h} + \epsilon_t \tag{9}$$

where matrix $X = (\beta_1, \beta_2, \beta_3)$ for Diebold-Li framework and matrix $X = (bs_1, bs_2, bs_3, bs_4, bs_5)$ for B-spline model

## 4. Data

In order to forecast quarterly macroeconomic activity, quarterly zero-coupon bond yield nominal spot rate in UK bond market data from fourth quarter of 1979 to the fourth quarter of 2009 is used in our research. For simplicity we fix maturities to 3, 6, 9, 12, 15, 18, 21, 24, 30, 36, 42, 48, 54, 60, 72, 84, 96, 108, 120, 144, 180 months. There 121 observations in total. The descriptive statistic for the real yields is given in Table 1. The real yield curves are wave like. The long rates are less volatile and more persistent than the short ones.

The UK Expenditure Approach Total GDP at Constant Prices, Seasonal Adjustment is used as the real GDP. The recession variable is using NBER definition which is negative real GDP growth for at least two quarters and plot 1 when the period is experiencing recession, otherwise plot 0. All data are collected from Thomson Reuters ECOWIN[4].

## 5. Results

### *5.1. Yield curve fitting*

The first stage of the forecasting is to construct the yield curve. We run 121 regressions using both Diebold-Li framework and B-spline model. All the results show that the modeled yield curves fit the original data well. However, B-spline framework shows a better performance. (see Figure 3 and 4 and Table 2)

### *5.2. Recession forecasting*

The results of probit model in-sample test are presented in Table 3 and Table 4. Pseudo $R^2$ is a value that is similar to the $R^2$ in the OLS representing the fitting ability of the model. Both Tables review that 3-quarter ahead forecasts have the best Pseudo

[4]Please see Appendix I for more information.

$R^2$ and lowest AIC, and all the coefficients of independent variables are significant. Therefore, it can be interpreted that the term structure explains the recession best by 3-quarter ahead, and all the independent variables can explain the recessions. Long-term and mid-term factors have a negative relationship with recession while short-term has a positive relationship with the recession. This is consistent with the fact that when there is a recession looming people tend to sell short-term bond and change it into mid- or long-term bonds.

Figure 5 contains the graphs showing predicted probability among Diebold-Li framework, B-spline model and spread model. B-spline captures all the three recessions while Diebold-Li framework captures the first and the last ones but not completely the second one by showing a probability not high enough. Spread model only captures the first two recessions. Moreover, the spread model also has a false alarm in the beginning of 2000. Figure 6 presents the comparison of the out-of-sample forecasting results by using these models. Both Diebold-Li framework and B-spline forecast the recent financial crisis successfully 3-quarter ahead. However, the spread model fails to forecast the recession by giving out a very low probability (smaller than 0.1) of recession when we are experiencing the financial crisis. It is worth mentioning that both Diebold-Li framework and B-spline model show a reducing probability of recession in 2009q3 while in reality the UK was still experiencing the recession.

### *5.3. GDP growth forecasting*

#### *5.3.1. In-sample test*

By using real GDP growth as a dependent variable, we firstly constructed Model A with both Diebold-Li method and B-spline method. The in-sample test results are given in Table 5 and Table 6. It is clear to see that both models show their best performance in 2-quarter ahead forecasting based on adjusted $R^2$. Actually from 1-quarter ahead to 4-quarter ahead they both show great forecasting ability. In Diebold-Li framework all the dependent variables are significant in the 2-quarter ahead model. This shows that all three parts of the yield curve contribute the explanation of the real GDP growth. The adjusted $R^2$ here can be interpreted as the percentage with which we can explain the GDP growth by using the model. Thus 51% real GDP growth can be explained by yield curve based on Diebold-Li framework. While in B-spline model 2-quarter ahead model explains 56% real GDP growth, which is a very good result compared to the yield spread, which can only explain a 22% real GDP growth base on the result of 5-quarter ahead modelling (see Table 10). This confirms that by including more useful information into the model using the whole yield curve inproves the forecasting ablility. Coefficients of short-term factor in 1- to 7-quarter ahead Deilbold-Li framwork based Model A are significant (see Table 5) suggests that monetary policy is a important part of the real growth. While in Deilbold-Li framwork based Model B, coefficients of short-term factor in 1- to 6- ahead forecast are significant suggest the same( see Table 7). The results from Model B (see Table 7 and Table 8) are very satisfactory based on both their $R^2$ and AIC. One quarter ahead forecasting base on both Diebold-Li framework and B-spline model get the best across the 1 to 8 quarter ahead forecasting horizons. Table 9 examine the models performace base on AIC. According to the table, Model B based on Deibold-Li framework 1-quarter ahead forecast is the best fitting model from all. From 1- to 4- quarter ahead forecasts Model B are better than Model A, while from 5- to 8-quarter ahdead forecasts Model A are better. In most cases, models based on B-spline framework achieve better performace than the ones base on Diebold-Li framwork.

Figure 7 shows the graph of models fitting chosen from the best fit of each model and we can compare with the result from forecasting using yield spread (see Figure 8). The comparison shows that whole yield curve fits the real growth much better than the yield spread.

*5.3.2. Out-of-sample test*

A good forecasting model should not only fit well in-sample but also predict well out-of-sample. We did out-of-sample test using data from 1979q4 to the time that the forecast is made, beginning in 1999q3 and extending through 2009q4. And in the mean time we compare the result with the result from classic yield spread forecasting model. We follow the literature by using 3-month and 10-year government bonds to calculate the yield spread. The forecast equation is:

$$\Delta Y_t = \alpha + \beta spread_{t-h} + \epsilon_t \tag{10}$$

Where $h$ is the forecasting horizon.

The results are shown in Table 11. RMSE (Root Mean Square Error) is a conventional tool to measure the efficiency of a forecast model in out-of-sample test. It is calculated as follows:

$$\text{RMSE} = \sqrt{\frac{\sum_{i=1}^{T}(A_i - F_i)^2}{T}} \tag{11}$$

Where $A$ is the actual value, and $F$ is the forecast value.

According to the results, it is important to note that Model B 1-quarter ahead bases on both Diebold-Li framework and B-splines get the smallest RMSE which makes it the best forecast model from all. Generally speaking, all the models show a impressive out-of-sample forecasting ability across all of the forecasting horizons. Another point from both forecast ability and fitting ability sight, models based on whole curve beat those based on the yield spread. Model B shows better fitting ability and forecasting performance as well as average stability in all the horizon base on $\bar{R}^2$ and RMSE, especially for 1- and 2-quarter ahead forecasting. This suggests that adding the term $\delta y_{t-h}$ to the model improves model's forecasting performance. Models based on B-spline framework get slightly better RMSE and better fitting ability.

Now we are comparing our out-of-sample results with HM Treasury forecast. In the UK, the official forecast is the one published by HM Treasury, and HM Treasury does 1,11 and 12 month-ahead forecasts. Because our forecasts are quarterly ahead, the only comparison we can do is 12-month as well as 4-quarter ahead. The results are reviewed in Table 12. This League Table presents that all the models based on the whole yield curve come out with similar results with HM Treasury, and these models outperform the spread model.

## 6. Conclusion

This paper proposes a model on the term structure forecasting output growth and recessions in the UK by using the whole yield curve rather than just the yield spread. By using Diebold-Li framework to get short-, mid- and long-term factors as yield curve variable and B-spline technique to form a yield curve other than traditional yield spread to forecast recession and economic growth, we get satisfactory results. According to the research there is strong evidence that in terms of recession forecasting Diebold-Li framework does better than the yield spread and B-spline model does even better(see Fig 5). Secondly, from in-sample test results, short-term yields which represent monetary policy are acting very important roles in real GDP growth forecasting in Model A and B.

Another finding of this paper is that models based on Diebold-Li and B-spline framework fit the yield curve better than the yield spread model and show a good forecasting ability in short horizon forecast especially 2-quarter ahead for Model A, 1-quarter ahead forecasting for Model B. From out-of-sample tests Model B based on both Diebold-Li framework and B-spline framework achieves very satisfactory results and show a stable forecasting ability in all forecasting horizons. All models based on the whole yield curve beat the results from models based on yield spread. From the comparison, it is important to note that Model B based on both B-spline generates closest forecast results to the one from HM Treasury.

Evidence found in this paper can be used by the government to improve both their recession forecasting and output growth forecasting model, and adjust their policies timely and efficiently.

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**Appendix I: Data source**

Here is the data source code from ECOWIN.

ew:gbr01021 United Kingdom, Expenditure Approach, Gross Domestic Product, Total, Constant Prices, GBP, 2005 CHND PRC

ew:gbr40312 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 3 month (0.25 year), Yield, GBP

ew:gbr40315 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 6 month (0.50 year), Yield, GBP

ew:gbr40318 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 9 month (0.75 year), Yield, GBP

ew:gbr40321 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 12 month (1.00 year), Yield, GBP

ew:gbr40324 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 15 month (1.25 year), Yield, GBP

ew:gbr40327 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 18 month (1.50 year), Yield, GBP

ew:gbr40330 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 21 month (1.75 year), Yield, GBP

ew:gbr40333 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 24 month (2.00 year), Yield, GBP

ew:gbr40339 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 30 month (2.50 year), Yield, GBP

ew:gbr40345 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 36 month (3.00 year), Yield, GBP

ew:gbr40351 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 42 month (3.50 year), Yield, GBP

ew:gbr40357 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 48 month (4.00 year), Yield, GBP

ew:gbr40363 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 54 month (4.50 year), Yield, GBP

ew:gbr40369 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 60 month (5.00 year), Yield, GBP ew:gbr40381 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 6.0 year, Yield, GBP

ew:gbr40383 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 7.0 year, Yield, GBP

ew:gbr40385 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 8.0 year, Yield, GBP

ew:gbr40387 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 9.0 year, Yield, GBP

ew:gbr40389 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 10.0 year, Yield, GBP

ew:gbr40399 United Kingdom, Zero Coupon Yields, Nominal, Spot Rate, 15.0 year, Yield, GBP

## Tables

Table 1: Descriptive statistics, yield curves.

| Maturity(Months) | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| 3 | 121 | 7.936 | 3.723 | 0.435 | 17.029 |
| 6 | 121 | 7.777 | 3.521 | 0.410 | 15.129 |
| 9 | 121 | 7.719 | 3.414 | 0.496 | 14.795 |
| 12 | 121 | 7.710 | 3.353 | 0.665 | 14.627 |
| 15 | 121 | 7.718 | 3.303 | 0.878 | 14.563 |
| 18 | 121 | 7.732 | 3.263 | 1.077 | 14.530 |
| 21 | 121 | 7.749 | 3.231 | 1.221 | 14.513 |
| 24 | 121 | 7.768 | 3.206 | 1.368 | 14.605 |
| 30 | 121 | 7.805 | 3.166 | 1.644 | 14.804 |
| 36 | 121 | 7.8386 | 3.140 | 1.891 | 14.950 |
| 42 | 121 | 7.870 | 3.122 | 2.107 | 15.047 |
| 48 | 121 | 7.898 | 3.112 | 2.298 | 15.107 |
| 54 | 121 | 7.925 | 3.106 | 2.467 | 15.138 |
| 60 | 121 | 7.949 | 3.104 | 2.619 | 15.149 |
| 72 | 121 | 7.989 | 3.105 | 2.883 | 15.135 |
| 84 | 121 | 8.018 | 3.108 | 3.108 | 15.100 |
| 96 | 121 | 8.033 | 3.109 | 3.303 | 15.053 |
| 108 | 121 | 8.036 | 3.103 | 3.473 | 15.005 |
| 120 | 121 | 8.026 | 3.090 | 3.624 | 14.955 |
| 144 | 121 | 7.975 | 3.042 | 3.880 | 14.847 |
| 180 | 121 | 7.840 | 2.933 | 4.053 | 14.646 |

Note: The table summarizes the general information of the data set we use, which is real zero-coupon rate in sample period 1979q4-2009q4.

Table 2: Comparison of estimation of the yield curve based on Diebold-Li and B-spline frameworks.

| Variable | Obs | Mean | Std. Dev. | Min | Max |
|---|---|---|---|---|---|
| | | Diebold-Li framework | | | |
| Adj $R^2$ | 121 | 0.88 | 0.191 | 0.005 | 0.97 |
| | | B-spline framework | | | |
| Adj $R^2$ | 121 | 0.96 | 0.002 | 0.96 | 0.97 |

Table 3: Probit model forecasting recession for Diebold-Li framework.

| qrt-ahead | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | constant | Pseudo $R^2$ | AIC |
|---|---|---|---|---|---|---|
| 1 | -0.397* | -0.235* | -0.136 | -0.107 | 0.36 | 65.53 |
| 2 | -0.546* | -0.010 | -0.323* | 0.212 | 0.49 | 53.61 |
| 3 | -0.998* | 0.336* | -0.439* | 1.192 | 0.60 | 43.97 |
| 4 | -0.444* | 0.324* | -0.280* | -0.208 | 0.47 | 53.82 |
| 5 | -0.280* | 0.522* | -0.257 | -1.005* | 0.48 | 50.02 |
| 6 | -0.119 | 0.568* | -0.172 | -1.491* | 0.42 | 52.55 |
| 7 | -0.039 | 0.582* | -0.124 | -1.760* | 0.40 | 51.25 |
| 8 | -0.010 | 0.613* | 0.0545 | -1.935* | 0.43 | 46.69 |

Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here $\alpha_1$ is coefficient of $b_1$, $\alpha_2$ is coefficient of $b_2$ and $\alpha_3$ is coefficient of $b_3$.

Table 4: Probit model forecasting recession for B-spline.

| qrt-ahead | $\alpha_{bs1}$ | $\alpha_{bs2}$ | $\alpha_{bs3}$ | $\alpha_{bs4}$ | $\alpha_{bs5}$ | constant | Pseudo $R^2$ | AIC |
|---|---|---|---|---|---|---|---|---|
| 1 | -0.105* | -0.573* | 0.998* | -0.721* | -0.077 | -0.802* | 0.48 | 58.41 |
| 2 | -0.067 | -0.752* | 0.934 | -0.712* | 0.008 | -0.865* | 0.63 | 45.12 |
| 3 | 0.012 | -0.475 | 1.424 | -2.169* | -0.372 | 1.025 | 0.79 | 31.04 |
| 4 | 0.039 | -0.153 | 0.606 | -0.865* | -0.104 | -0.800 | 0.57 | 49.27 |
| 5 | 0.101* | 0.181 | -0.161 | -0.581 | 0.175 | -1.377* | 0.54 | 48.87 |
| 6 | 0.104* | 0.338 | -0.115 | -0.630 | 0.167 | -1.814* | 0.49 | 51.47 |
| 7 | 0.109* | 0.225 | 0.029 | -0.395 | -0.027 | -1.912* | 0.43 | 53.55 |
| 8 | 0.121 * | 0.526 | -0.239 | -0.364 | -0.056 | -1.932* | 0.43 | 50.65 |

Note: $\alpha_x$ represents the coefficient of variable $x$. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 5: Model A based on Diebold-Li framework.

| qrt-ahead | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | constant | $R^2$ | AIC |
|---|---|---|---|---|---|---|
| 1 | 0.006* | 0.002* | 0.002* | -0.005 | 0.49 | -636.7 |
| 2 | 0.007* | 0.002* | 0.002* | $-0.007^*$ | 0.52 | -637.8 |
| 3 | 0.006* | 2.21E-04 | 0.001* | -0.004 | 0.46 | -624.6 |
| 4 | 0.005* | -0.001 | 0.001 | 7.77E-05 | 0.37 | -604.4 |
| 5 | 0.003* | $-0.003^*$ | 0.0004 | 0.006 | 0.29 | -592.8 |
| 6 | 0.002* | $-0.003^*$ | 8.16E-05 | 0.011* | 0.23 | -585.9 |
| 7 | 0.001* | $-0.004^*$ | -6.61E-06 | 0.014* | 0.21 | -580.8 |
| 8 | 0.001 | $-0.003^*$ | 4.0E-04 | 0.015* | 0.16 | -572.4 |

Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here $\alpha_1$ is coefficient of $b_1$, $\alpha_2$ is coefficient of $b_2$ and $\alpha_3$ is coefficient of $b_3$.

Table 6: Model A based on B-spline framework.

| qrt-ahead | $\alpha_{bs1}$ | $\alpha_{bs2}$ | $\alpha_{bs3}$ | $\alpha_{bs4}$ | $\alpha_{bs5}$ | constant | $R^2$ | AIC |
|---|---|---|---|---|---|---|---|---|
| 1 | 5.75E-04* | 0.005* | -0.004 | 0.005* | 7.13E-04 | -0.003 | 0.52 | -639.7 |
| 2 | 4.35E-04* | 0.004* | -0.004 | 0.007* | 0.001 | -0.005 | 0.56 | -643.9 |
| 3 | 2.48E-04* | 0.002* | -0.005 | 0.008* | 9.95E-04 | -0.001 | 0.51 | -632.8 |
| 4 | 6.47E-05 | 1.7E-04 | -0.005 | 0.010* | 8.89E-04 | 0.003 | 0.44 | -614.5 |
| 5 | -1.4E-04 | -0.001 | -0.005 | 0.010* | 9.58E-04 | 0.009* | 0.36 | -600.1 |
| 6 | -2.4E-04 | -0.002 | -0.005 | 0.009* | 7.55E-04 | 0.014* | 0.30 | -591.8 |
| 7 | -4E-04 | -0.002 | -0.004 | 0.008* | 0.001 | 0.017* | 0.25 | -583.0 |
| 8 | -5E-04 | -0.003 | -0.003 | 0.007* | 0.001 | 0.018* | 0.20 | -571.0 |

Note: $\alpha_x$ represents the coefficient of variable $x$. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 7: Model B based on Diebold-Li framework.

| qrt-ahead | $\phi$ | $\alpha_1$ | $\alpha_2$ | $\alpha_3$ | constant | $R^2$ | AIC |
|---|---|---|---|---|---|---|---|
| 1 | 0.810* | 0.002* | -3.2E-05 | 3.9E-04 | -0.004* | 0.842 | -776.4 |
| 2 | 0.630* | 0.004* | -0.00021 | 5.4E-04 | -0.008* | 0.717 | -699.1 |
| 3 | 0.465* | 0.004* | -0.00084 | 5.0E-04 | -0.005* | 0.558 | -647.1 |
| 4 | 0.255* | 0.004* | -0.00172 | 5.1E-04 | -0.001* | 0.398 | -607.8 |
| 5 | 0.159 | 0.003* | -0.003 | 1.3E-04 | 0.00567 | 0.302 | -592.7 |
| 6 | 0.093 | 0.002* | -0.003* | -7.6E-05 | 0.011* | 0.238 | -584.6 |
| 7 | -0.012 | 0.002 | -0.004* | 1.38E-05 | 0.015* | 0.212 | -578.8 |
| 8 | -0.121 | 0.002 | -0.003* | -0.00018 | 0.017* | 0.192 | -571.4 |

Note: * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times. Here $\phi$ is coefficient of lag of gdp growth, $\alpha_1$ is coefficient of $b_1$, $\alpha_2$ is coefficient of $b_2$ and $\alpha_3$ is coefficient of $b_3$.

Table 8: Model B based on B-spline framework.

| qrt-ahead | $\phi$ | $\alpha_{bs1}$ | $\alpha_{bs2}$ | $\alpha_{bs3}$ | $\alpha_{bs4}$ | $\alpha_{bs5}$ | constant | $R^2$ | AIC |
|---|---|---|---|---|---|---|---|---|---|
| 1 | 0.797* | 3.6E-05 | 1.9E-04 | -2.9E-04 | 2.2E-03 | -1.4E-05 | -3.6E-03 | 0.845 | -774.7 |
| 2 | 0.598* | 7.8E-05 | 4.5E-04 | -2.0E-03 | 0.005* | 6.6E-04 | -0.006* | 0.728 | -700.1 |
| 3 | 0.411* | 4.4E-05 | -1.4E-04 | -2.7E-03 | 0.007* | 4.9E-04 | -0.003 | 0.584 | -650.3 |
| 4 | 0.169 | -6.1E-06 | -5.4E-04 | -4.5E-03 | 0.009* | 6.4E-04 | -0.003 | 0.452 | -614.9 |
| 5 | 0.075 | -1.7E-04 | -1.5E-03 | -4.9E-03 | 0.010* | 8.4E-04 | 0.009* | 0.359 | -598.5 |
| 6 | 0.011 | -2.4E-04 | -2.0E-03 | -4.9E-03 | 0.009* | 7.4E-04 | 0.014* | 0.296 | -589.8 |
| 7 | -0.091 | -3.6E-04 | -1.8E-03 | -4.8E-03 | 0.009* | 1.2E-03 | 0.018* | 0.258 | -581.6 |
| 8 | -0.189 | -4.4E-04 | -1.8E-03 | -4.1E-03 | 0.007* | 1.9E-03 | 0.019* | 0.221 | -571.5 |

Note: $\alpha_x$ represents the coefficient of variable $x$. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 9: Model comparison based on AIC

| | Model A | | Model B | |
|---|---|---|---|---|
| qrt-ahead | DL | BS | DL | BS |
| 1 | -636.7 | -639.7 | -776.4 | -774.4 |
| 2 | -637.8 | -643.9 | -699.1 | -700.1 |
| 3 | -624.6 | -632.8 | -647.1 | -650.3 |
| 4 | -604.4 | -614.5 | -607.8 | -614.9 |
| 5 | -592.8 | -600.1 | -592.7 | -598.5 |
| 6 | -586.0 | -591.8 | -584.6 | -589.8 |
| 7 | -580.8 | -583.0 | -578.8 | -581.6 |
| 8 | -572.3 | -571.0 | -571.4 | -571.5 |

Note: AIC is short for Akaike information criterion. Selection rules: The lower AIC the model has, the better it fits the data. DL = Deibold-Li framework, BS = B-spline framework.

Table 10: Results based on spread model

| qrt-ahead | $\alpha_1$ | constant | $R^2$ |
|---|---|---|---|
| 1 | -0.002 | 0.0201* | 0.01 |
| 2 | -0.004* | 0.021* | 0.07 |
| 3 | -0.005* | 0.021* | 0.12 |
| 4 | -0.006* | 0.021* | 0.20 |
| 5 | -0.007* | 0.022* | 0.22 |
| 6 | -0.006* | 0.023* | 0.19 |
| 7 | -0.006* | 0.023* | 0.18 |
| 8 | -0.005* | 0.023* | 0.14 |

Note: $\alpha_x$ represents the coefficient of variable $x$. * is significant in 95% confidence level with coefficients' standard error bootstrapped 1000 times.

Table 11: Out-of-sample test results.

| qrt-ahead | | | Model A | | | | Model B | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Spread | | D-L | | B-spline | | D-L | | B-spline | |
| | RMSE | $\bar{R}^2$ | RMSE | $\bar{R}^2$ | RMSE | $\bar{R}^2$ | RMSE | $\bar{R}^2$ | RMSE | $\bar{R}^2$ |
| 1 | 0.029 | 0.13 | **0.020** | 0.47 | **0.020** | 0.51 | **0.009** | **0.79** | **0.009** | **0.79** |
| 2 | 0.028 | 0.21 | 0.02 | **0.54** | 0.02 | **0.59** | 0.015 | 0.71 | 0.015 | 0.72 |
| 3 | 0.027 | 0.27 | 0.022 | 0.5 | 0.021 | 0.57 | 0.020 | 0.61 | 0.020 | 0.64 |
| 4 | 0.026 | 0.32 | 0.023 | 0.44 | 0.023 | 0.53 | 0.023 | 0.48 | 0.023 | 0.55 |
| 5 | **0.026** | **0.33** | 0.025 | 0.38 | 0.025 | 0.48 | 0.025 | 0.42 | 0.024 | 0.50 |
| 6 | 0.026 | 0.29 | 0.026 | 0.33 | 0.026 | 0.43 | 0.026 | 0.37 | 0.025 | 0.45 |
| 7 | 0.027 | 0.27 | 0.027 | 0.3 | 0.027 | 0.38 | 0.026 | 0.31 | 0.025 | 0.38 |
| 8 | 0.028 | 0.22 | 0.027 | 0.24 | 0.028 | 0.32 | 0.025 | 0.24 | 0.025 | 0.32 |

Note: D-L represents Diebold-Li framework. Selection rules: Choose the model with smallest RMSE and biggest $\bar{R}^2$. In-sample period:1979q4-1999q4, out-of-sample period 2000q1-2009q4; RMSE: Root Mean Square Error is computed as in equation (11).

Table 12: League Table of models out-of-sample 4-quarter ahead forecasting.

| Forecastor | RMSE | Performance |
|---|---|---|
| HM Treasury forecast | 0.0222 | Best |
| Model B B-spline | 0.0229 | ↓ |
| Model B Diebold-Li | 0.0231 | ↓ |
| Model A B-spline | 0.0231 | ↓ |
| Model A Diebold-Li | 0.0236 | ↓ |
| Spread model | 0.0262 | Worst |

Note: Selection rule: choose the model with the smallest RMSE. In-sample period: 1979q4-1999q4, out-of-sample period: 2000q1-2009q4.

## Figures

Figure 1: Loading figure for Diebold and Li framework.

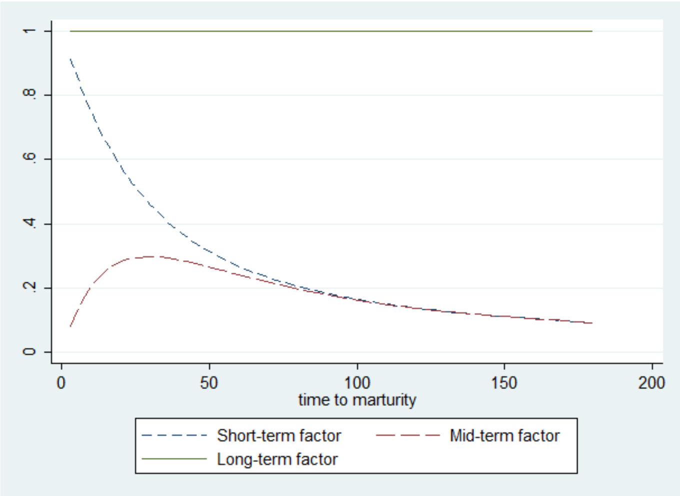



Figure 2: Loading figure for B-spline framework.

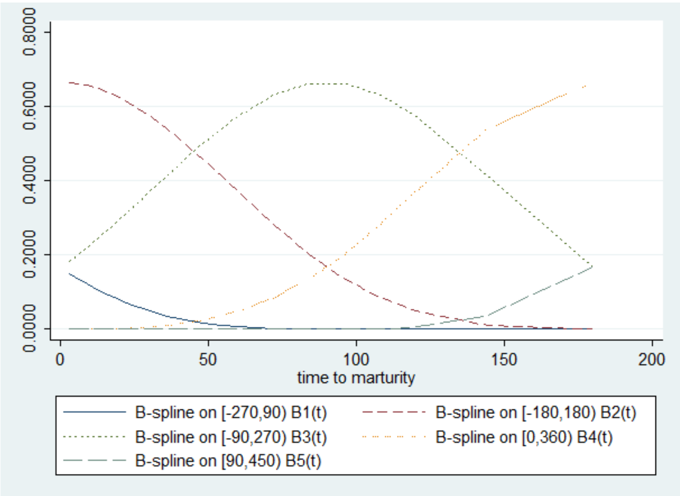
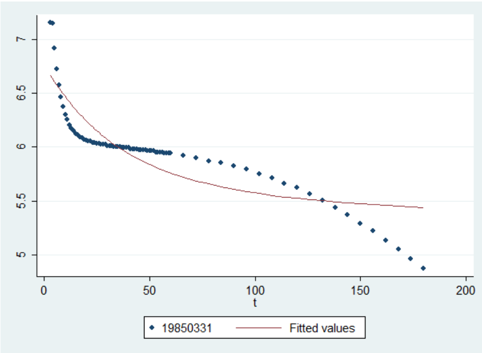
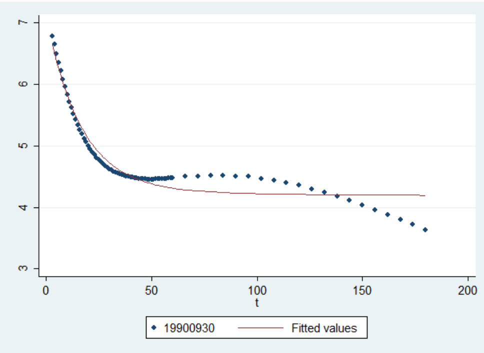


Figure 3: Diebold-Li Yield Curve Fitted for selected dates.

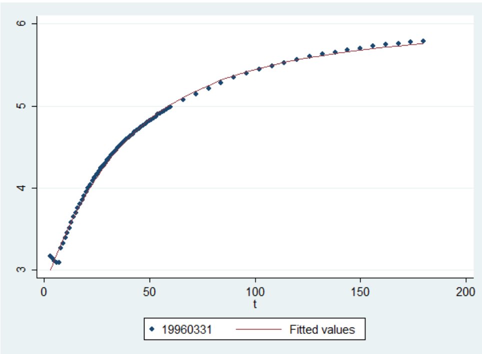
(a) The yield curve and fitting on MAR 31,1985




(b) The yield curve and fitting on SEP 30,1990




(c) The yield curve and fitting on MAR 31,1996




(d) The yield curve and fitting on SEP 30,2001

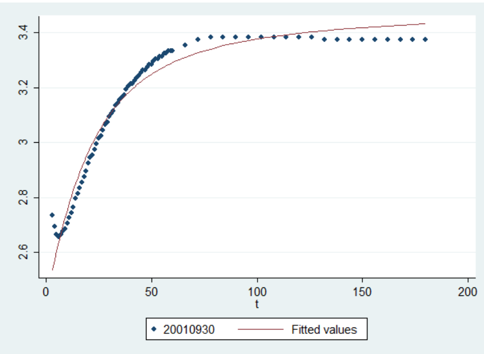
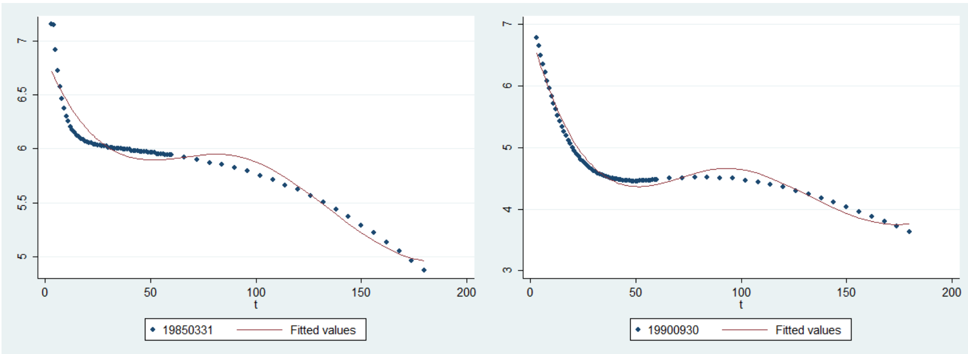


Figure 4: B-spline Yield Curve Fitted for selected dates.

(a) The yield curve and fitting on MAR 31,1985

(b) The yield curve and fitting on SEP 30,1990




(c) The yield curve and fitting on MAR 31,1996

(d) The yield curve and fitting on SEP 30,2001

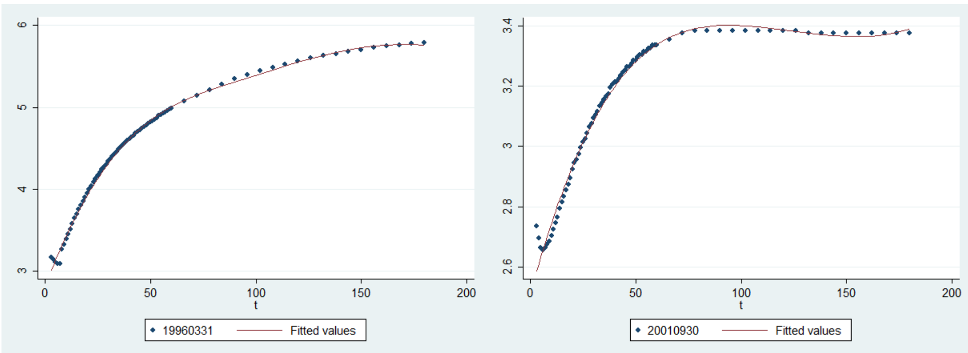
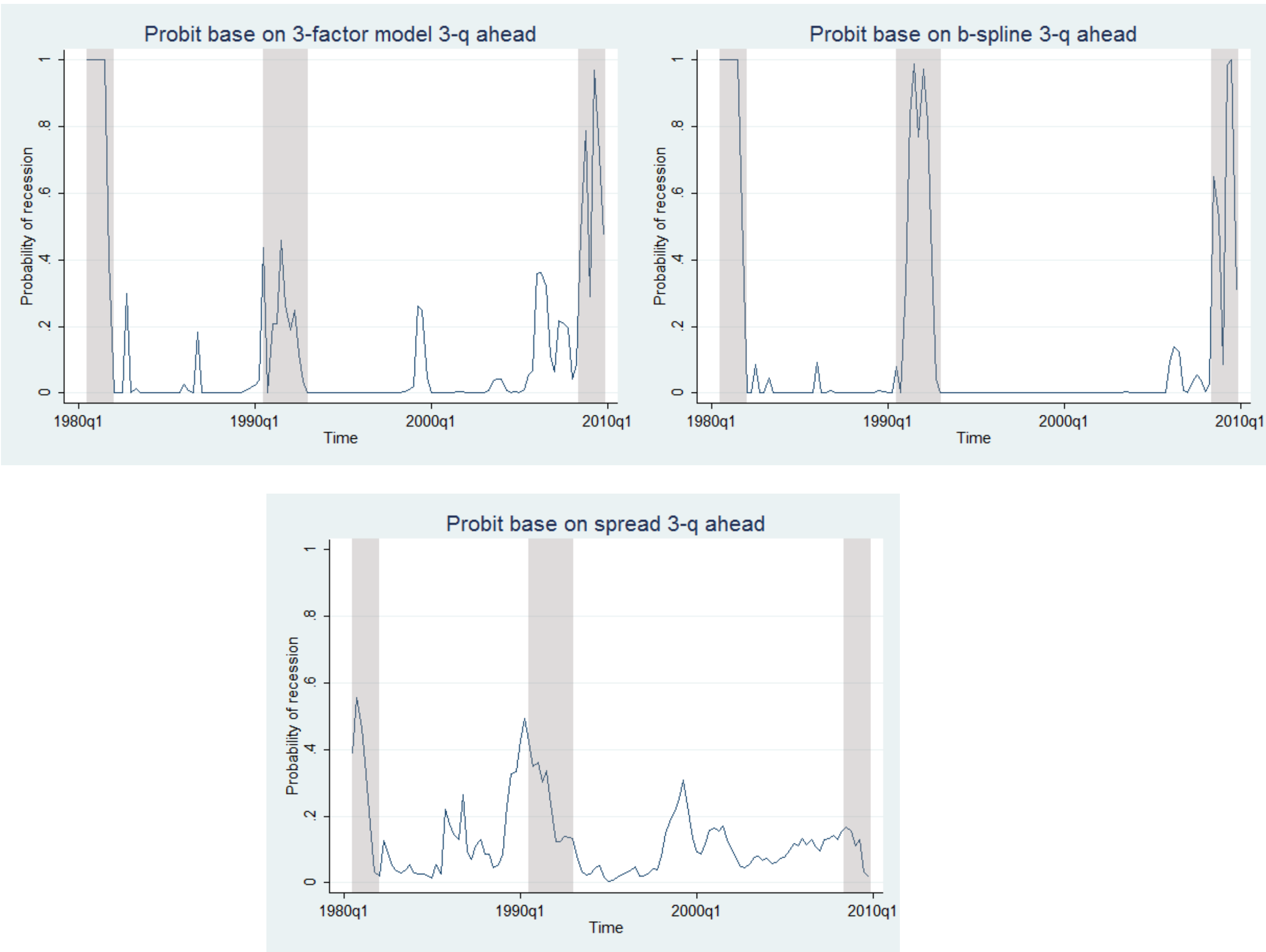
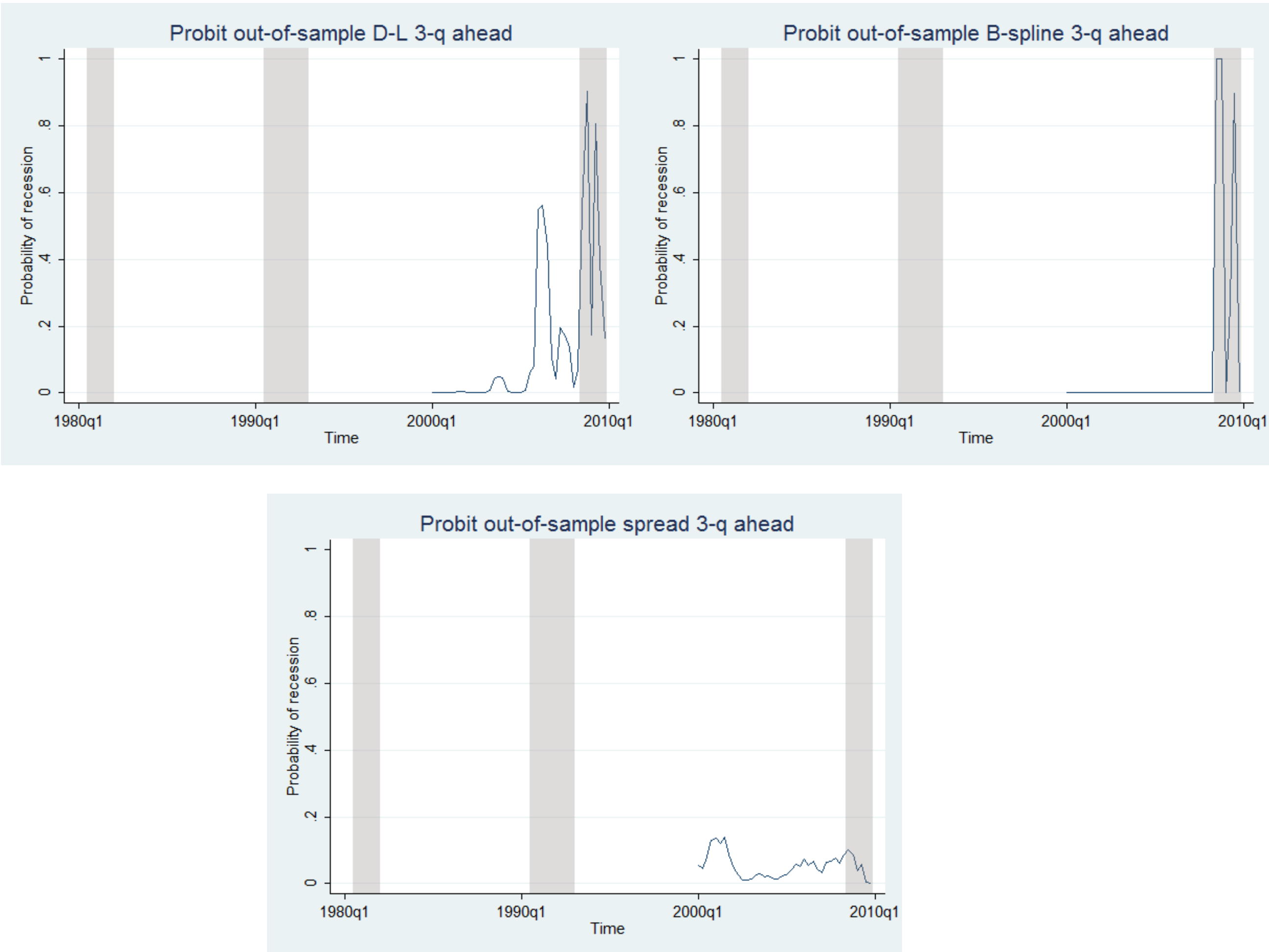


Figure 5: Comparison of model fitting using probit models 3-quarter ahead.




Note: Shades area are recessions under the NBER definition.

Figure 6: Comparison of model out-of-sample using probit models 3-quarter ahead.

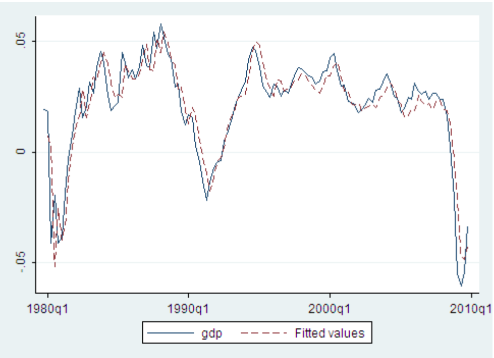



Note: Shades area are recessions under the NBER definition. In-sample period:1979q4-1999q4, out-of-sample period 2000q1-2009q4

Figure 7: In-sample test fitting results.

(a) 2q-ahead Diebold-Li Model A


(b) 1q-ahead Diebold-Li Model B




(c) 2q-ahead B-spline Model A


(d) 1q-ahead B-spline Model B

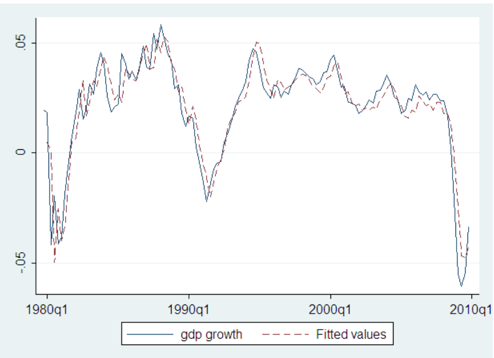


Figure 8: In-sample result from spread 2-q ahead for comparison.